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# Electric charges at rest Part-I

In this chapter we will learn how to calculate electric field for static charge distributions. Greeks were the first to observe electrical phenomena. They observed that a piece of amber when rubbed with fur or cat skin acquired the property of attracting small bits of paper and straw. Later on, Coulomb a French scientist conducted experiments using a torsion balance set up to study forces between charged objects.

In the following sections we will describe Coulomb's law and discuss about Gauss's law and then see how potential is used to calculate electric field. For complete grasp of the above topics reader is advised to read section 2.1 to section 2.18 continuously without breaking any section or example in between. After reading upto section 2.18 the reader will be able to appreciate why there are three different methods, Coulombs law, Gauss Law and potential to calculate electric field for a given charge distribution.

## 2.1 Coulomb's Law

Charles Augustin Coulomb, a French scientist using his torsion balance setup performed a series of experiments to study the nature of forces between two charged objects. The results obtained by Coulomb using his torsion balance setup is called Coulomb's law. As per Coulomb's law the force between two charges

i) is directly proportional to the product of their charges.

ii) is inversely proportional to the square of the distance between the two charges and acts along the line joining the two charges.

iii) the nature of the medium in which the charged objects are situated.

A source charge q is separated from the test charge Q by a distance $r$ as shown in figure 2.1.

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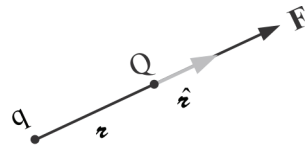


**Fig.2.1**

The force exerted by q on Q as per Coloumb's law is then

$$\mathbf{F} = k\frac{qQ}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.1)$$

Here $\hat{\boldsymbol{r}}$ is the unit vector in the direction of **F**. k is the proportionality constant whose value depends on the medium in which the charges are situated, units in which force, charge and distance are expressed. Because in figure 2.1 the charges are like charges the forces are repulsive in nature. If the charges are unlike charges the forces are attractive in nature.

In SI units force is measured in Newton, distance in metre and charge in Coulomb. The proportionality constant k is written as

$$k = \frac{1}{4\pi\varepsilon_o}$$

So that equation 2.1 is

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.2)$$

Here $\varepsilon_o$ is the absolute permittivity of free space whose value is given by

$$\varepsilon_o = 8.85\times10^{-12}\ C^2\ N^{-1}\ m^{-2}$$

If the charges are not situated in vacuum but in a medium of permitivity ε, then $\varepsilon = \varepsilon_o\varepsilon_r$. The force between the two charges is

$$\mathbf{F} = \frac{1}{4\pi\varepsilon}\frac{qQ}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.3)$$

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{qQ}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.4)$$

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Here $\varepsilon_r$ is the dielectric constant of the medium and is a dimensionless quantity. For air and vacuum $\varepsilon_r = 1$.

## 2.2 Electric Field Intensity

The electric field intensity **E** is defined as the force exerted on a unit positive charge placed at that point in an electrostatic field. Here electrostatic field refers to the space surrounding an electrostatic charge where the electrostatic forces can be detected.

The electric field produced by charge q in figure 2.1 is

$$\mathbf{E} = \frac{\mathbf{F}}{Q} \qquad ...(2.5)$$

Substituting equation 2.2 in equation 2.5

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.6)$$

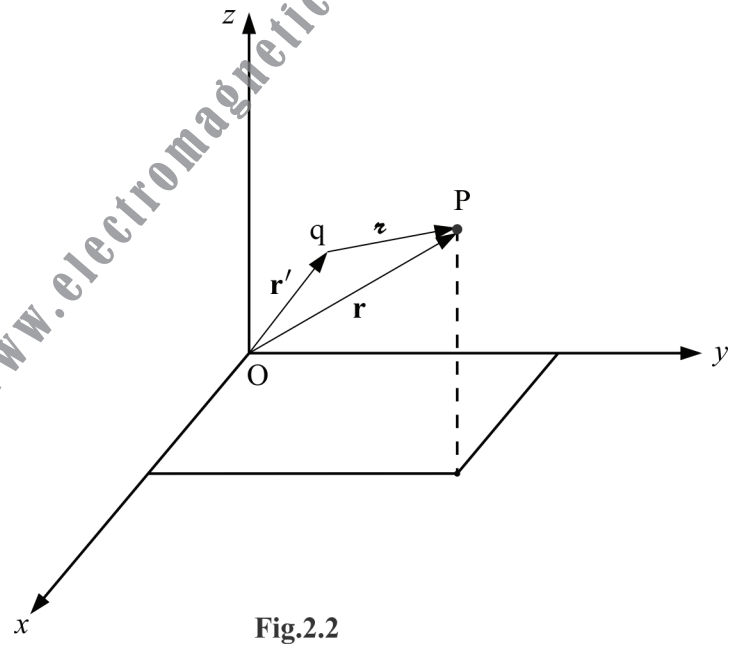


**Fig.2.2**

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Consider figure 2.2. Point P is situated at a distance of r from the origin and charge q is situated at a distance of $r'$ from origin. The distance between q and point P is $\boldsymbol{\mathcal{r}}$. The electric field at point P is then

$$\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\mathcal{r}^2}\hat{\boldsymbol{\mathcal{r}}} \qquad ...(2.7)$$

where $\boldsymbol{\mathcal{r}} = \mathbf{r} - \mathbf{r}'$ .

## 2.3 Electric Field Intensity Due to a Group of Discrete Point Charges

Consider a group of discrete point charges $q_1$, $q_2$, $q_3$.....$q_n$ as shown in figure 2.3 exerting force on test charge Q. We are interested in calculating the force exerted by the charges $q_1$, $q_2$, $q_3$.....$q_n$ on the test charge Q. The net force acting on charge Q is

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + ..... + \mathbf{F_n} \qquad ...(2.8)$$

$$\mathbf{F} = \sum_{i=1}^{n} \mathbf{F_i} \qquad ...(2.9)$$

where the force $\mathbf{F}_i$ in the above equation is given by equation 2.3 replacing q by $q_i$, $\boldsymbol{\mathcal{r}}$ by $\boldsymbol{\mathcal{r}}_i$ and $\hat{\boldsymbol{\mathcal{r}}}$ by $\hat{\boldsymbol{\mathcal{r}}}_i$.

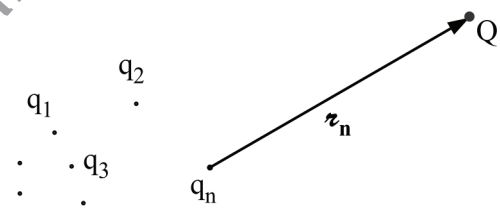


**Fig.2.3**

Thus the net force acting on the test charge Q is found by calculating the force due to individual charges separately and then summing all the individual forces. However the addition is vector addition not a scalar addition. That is forces in equation 2.8 must be vectorically added [see section 1.1].



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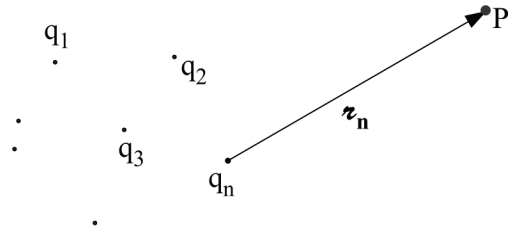


**Fig.2.4**

Now consider figure 2.4. We are interested in calculating the net electric field at point P.

From equation 2.9

$$\frac{\mathbf{F}}{Q} = \sum_{i=1}^{n} \frac{\mathbf{F_i}}{Q} \qquad ...(2.10)$$

$$\Rightarrow \mathbf{E} = \sum_{i=1}^{n} \mathbf{E_i} \qquad ...(2.11)$$

Once again the sum in equation 2.11 is vector addition not scalar addition.

With the aid of equation 2.6, 2.11 can be written as

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i \qquad ...(2.12)$$

## 2.4 Continuous Charge Distributions

In the previous section we defined the electric field intensity due to a group of discrete point charges. Now let us consider charges that are spread over space. There are three different continuous charge distributions as shown in figure 2.5.

1) *Line Charge*: Charge distributed over a line as shown in figure 2.5a. Mathematically we describe the line charge with symbol λ

$$\lambda = \frac{dq}{dl'} \qquad ...(2.13)$$

where λ is charge per unit length.

2) *Surface Charge*: Charge distributed over a surface as shown in



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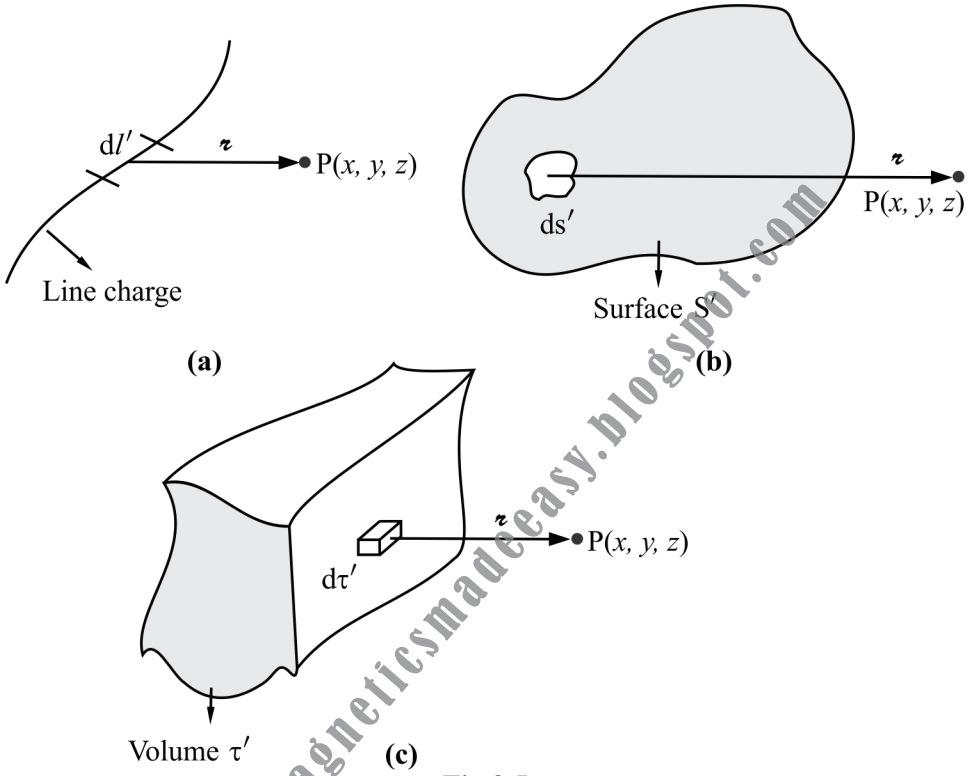


**Fig.2.5**

figure 2.5b. Mathematically we describe the surface charge with symbol $\sigma$

$$\sigma = \frac{dq}{ds'} \qquad ...(2.14)$$

where $\sigma$ is charge per unit area.

3) *Volume Charge*: Charge is distributed over a volume as shown in figure 2.5c. Mathematically we describe volume charge with symbol $\rho$

$$\rho = \frac{dq}{d\tau'} \qquad ...(2.15)$$

$\rho$ is charge per unit volume.



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For a small charge dq equation 2.6 can be written as

$$\mathbf{E}_{dq} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.16)$$

i) For a line charge $dq = \lambda dl'$. Substituting for dq in equation 2.16 and integrating over the entire line $l'$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\int_{l'}\frac{\lambda dl'}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.17)$$

ii) For a surface charge $dq = \sigma ds'$. Substituting for dq in equation 2.16 and integrating over the entire surface $S'$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\int_{S'}\frac{\sigma ds'}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.18)$$

iii) For a volume charge $dq = \rho d\tau'$. Substituting for dq in equation 2.16 and integrating over the entire volume $\tau'$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\int_{\tau'}\frac{\rho d\tau'}{r^2}\hat{\boldsymbol{r}} \qquad ...(2.19)$$

Equations 2.17, 2.18, 2.19 gives the electric field intensity at point P corresponding to the charge distributions shown in figure 2.5a, 2.5b, 2.5c, respectively.

Finally the total charge enclosed for the line charge is

$$q_{tot} = \int_{l'}\lambda dl' \qquad ...(2.20)$$

surface charge is

$$q_{tot} = \int_{S'}\sigma ds' \qquad ...(2.21)$$

and volume charge is

$$q_{tot} = \int_{\tau'}\rho\, d\tau' \qquad ...(2.22)$$

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In figure 2.5 we have used primed coordinates $dl'$, $ds'$, $d\tau'$ instead of $dl$, $ds$, $d\tau$. The above difference has been made in order to distinguish between source points where charge is located and field points where the field is calculated. For example in figure 2.5c within the volume $\tau'$ all the points will be denoted with prime coordinates i.e., $(x', y', z')$ because within the volume $\tau'$ all the charge is located. However for points like P in figure 2.5c we use unprimed coordinates $(x, y, z)$ to denote that it is the point where field is calculated. Thus we use primed symbols for source points and unprimed symbols for field points. One must be careful enough to integrate with respect to either source points or field points where ever necessary.

## 2.5 A Note About Coulomb's Law

Equations (2.12), (2.17), (2.18), (2.19) calculate electric field vector **E** directly in terms of vector. The reader can observe that $\hat{\boldsymbol{\imath}}$ is present in the equations mentioned above signifying that **E** is directly calculated in terms of a vector. We have already seen in section 1.1 and example 1.10 that vectors are very difficult to deal with. Hence, calculation of **E** using the above mentioned equations becomes complicated even for simple charge distributions. To circumvent this problem much easier methods to calculate **E** has been developed - Gauss's law and potential formulation. As we will see, in Gauss law we use symmetry of the problem to calculate **E** easily, while in potential formulation we reduce the vector problem into a scalar problem so that **E** is calculated easily. This point will be more clear to the reader when the reader completes section 2.18.

## 2.6 Calculating Electric Field E Using Coulomb's Law

Initially we will provide a note on calculating electric field **E** in two dimension in cartesian coordinates system. The points discussed here are valid in three dimensions and other coordinate systems. The electric field **E** in two dimensions in given by

$$\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}}$$

While solving problems using Coulomb's law it will be sufficient to calculate $E_x$ and $E_y$ for the given charge distribution and substitute the values in the above equation to get **E**. However in calculating **E** for certain charge distributions like



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the ones solved in example 2.4, 2.5, 2.7 either $E_x$ or $E_y$ might cancel out each other and the remaining component alone is to be calculated. But for certain charge distributions as shown in example 2.9, section 2.14 neither components will cancel out and hence $E_x$ and $E_y$ needs to be calculated separately. Whether the components will cancel out (or) get added up depends on the charge distribution and the point where the electric field is calculated. The above conclusions holds good for three dimensions and other coordinate systems.

**Example 2.1**

Two charges of magnitude $5\times10^{-9}$ Coulomb and $7\times10^{-9}$ Coulomb are separated by 25 cm in vacuum. Find the force of interaction between them. If the same charges are separated by the same distance in mica ($\varepsilon_r = 6$) what is the force of interaction?

***Solution:***

We know that

$$F = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_o}\frac{(5\times10^{-9})(7\times10^{-9})}{(25\times10^{-2})^2}$$

$$\text{But } \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi(8.85\times10^{-12})} = 9\times10^9$$

$$\text{Hence } F = \left(9\times10^9\right)\frac{(5\times10^{-9})(7\times10^{-5})}{(25)^2}$$

$$F = 0.504\times10^{-5} \text{ Newtons}$$

For Mica $\varepsilon_r = 6$

$$\text{Hence } F_{mica} = \frac{F}{\varepsilon_r} = \frac{0.504\times10^{-5}}{6}$$



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$F_{mica} = 8.4 \times 10^{-7}$ Newtons.

**Example 2.2**

A line charge $\lambda = 5(y')^4$ nano coulomb/m is situated along *y* axis from 0 to 7 cm. Calculate the total charge.

***Solution:***

From equation 2.20

$$q_{tot} = \int \lambda dl'$$

$$\Rightarrow q_{tot} = \int_0^7 5(y')^4 dy' = 5\left[\frac{(y')^5}{5}\right]_0^7$$

$$\Rightarrow q_{tot} = 7^5 \text{ nano coulombs}$$

$$\Rightarrow q_{tot} = 16.807 \ \mu \text{ coulomb}$$

**Example 2.3**

In a circular disk of radius R charge density is given by $\sigma = k(r')^2$. Calculate the total charge.

***Solution:***

From equation 2.21

$$q_{tot} = \int \sigma ds'$$

From figure 2.6

$$ds' = r' \, dr' \, d\phi'$$

Hence $q_{tot} = \int_0^R \int_0^{2\pi} k(r')^2 \, r' \, dr' \, d\phi'$

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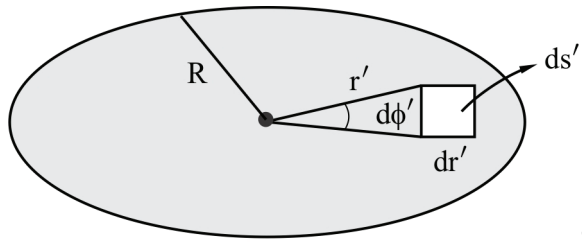


**Fig.2.6**

$$\Rightarrow q_{tot} = 2\pi k \frac{R^4}{4}$$

$$\Rightarrow q_{tot} = \frac{\pi k R^4}{2}$$

**Example 2.4**

Two charges each having magnitude q are separated by a distance 2a as shown in figure 2.7. Find the net electric field at point P.

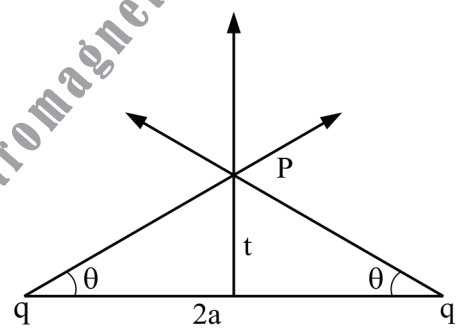


**Fig.2.7**

***Solution:***

Let each charge q produce an electric field of $\mathbf{E}_q$ at point P as shown in figure 2.8a.



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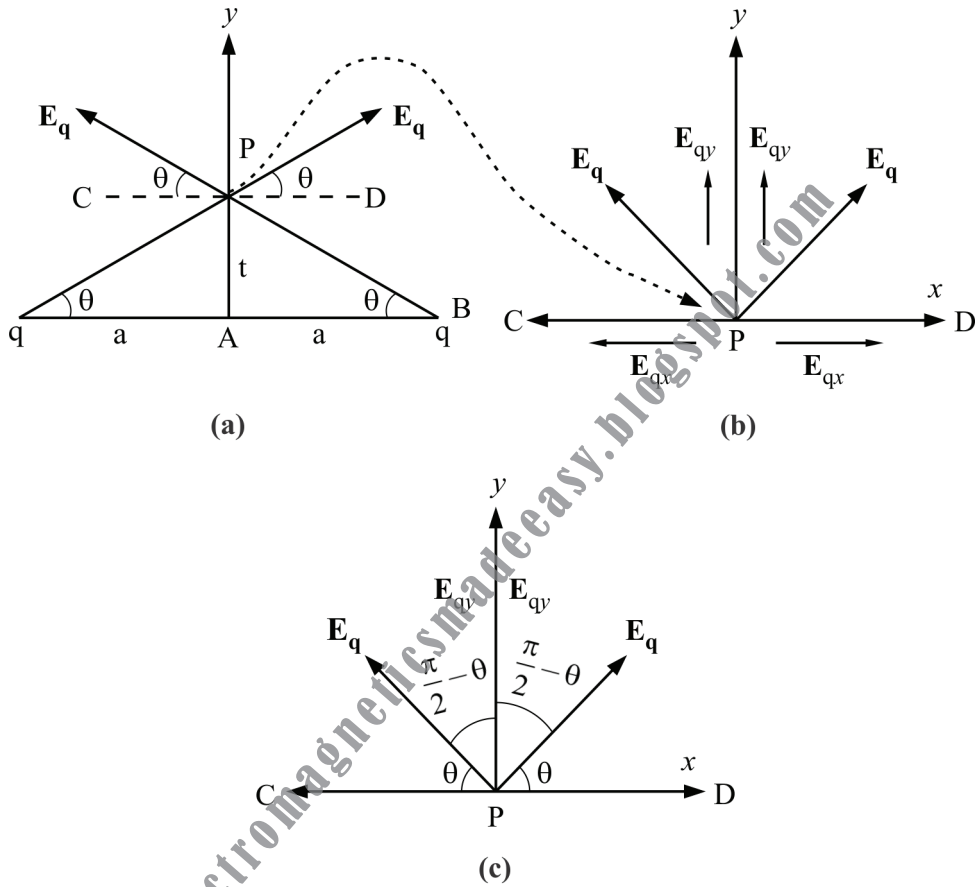


**Fig.2.8**

The magnitude of electric field $\mathbf{E_q}$ produced by q at point P is

$$|\mathbf{E_q}| = \frac{1}{4\pi\varepsilon_o}\frac{q}{PB^2}$$

The line CPD shown in figure 2.8a is shown separately in figure 2.8b. Resolving $\mathbf{E_q}$ into $\mathbf{E}_{qx}$ and $\mathbf{E}_{qy}$ in figure 2.8b we see that the $x$ components $\mathbf{E}_{qx}$ cancel out each other while $\mathbf{E}_{qy}$ components add up. Hence, the net electric field at point P by the two charges each of magnitude q is given by



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$$\mathbf{E} = 2E_{qy}\hat{\mathbf{j}}$$

From figure 2.8c

$$\cos\left(\pi/2 - \theta\right) = \frac{E_{qy}}{E_q}$$

$$\Rightarrow E_{qy} = E_q \sin\theta$$

Substituting for $E_{qy}$ in **E**

$$\mathbf{E} = 2E_q \sin\theta\,\hat{\mathbf{j}}$$

From figure 2.8a

$$\sin\theta = \frac{AP}{PB} = \frac{t}{\sqrt{t^2 + a^2}}$$

Substituting for sin θ in **E**

$$\mathbf{E} = 2\,E_q \frac{t}{\sqrt{t^2 + a^2}}\hat{\mathbf{j}}$$

Substituting for $E_q$

$$\mathbf{E} = 2\frac{1}{4\pi\varepsilon_o}\frac{q}{PB^2}\frac{t}{\sqrt{t^2 + a^2}}\hat{\mathbf{j}}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\frac{qt}{\left[t^2 + a^2\right]^{3/2}}\hat{\mathbf{j}} \quad \text{as} \quad PB^2 = t^2 + a^2$$

**Example 2.5**

A line charge is present in the region $-a \le y' \le a$ with a uniform line charge density λ as shown in figure 2.9. Calculate the electric field at point P at a distance t from point O, where O is the centre of the line charge?



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**Fig.2.9**

***Solution:***

In order to calculate the electric field produced by the entire line charge let us divide the line charge into number of small strips each of length $dl'$. Let us consider two strips $dy'$ each holding a charge $\lambda dy'$ as shown in figure 2.10. Let us calculate the electric field by these two strips initially and finally integrate between

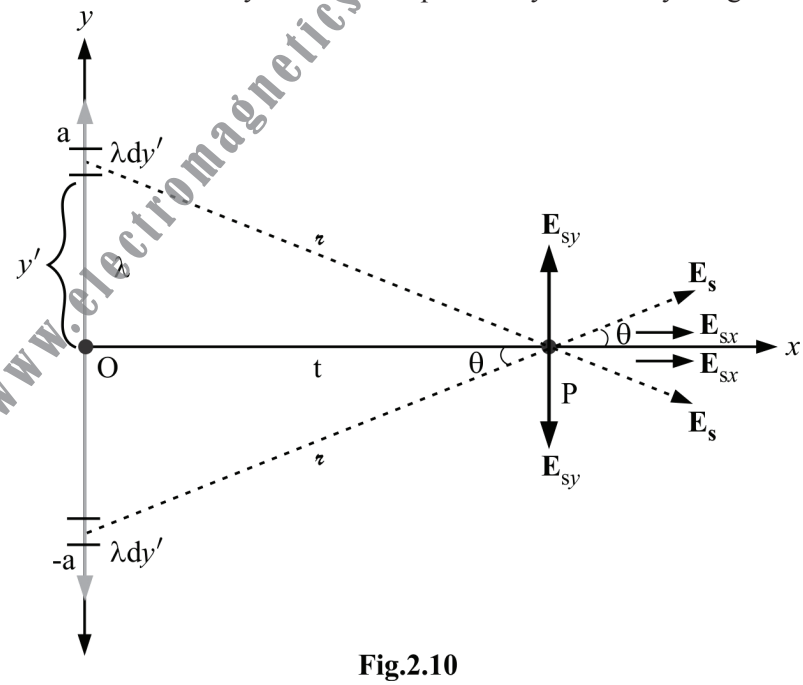


**Fig.2.10**



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0 to a in order to calculate the net electric field produced by the line charge lying between –a to a.

The magnitude of electric field produced by single strip $\lambda dy'$ at point P as shown in figure 2.10 is

$$|\mathbf{E_s}| = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dy'}{r^2}$$

$$|\mathbf{E_s}| = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dy'}{\left[(y')^2 + t^2\right]}$$

Resolving the electric field $\mathbf{E_s}$ due to the two strips into $\mathbf{E}_{sx}$ and $\mathbf{E}_{sy}$ components as shown in figure 2.10 we see that $\mathbf{E}_{sy}$ components cancel out each other while $\mathbf{E}_{sx}$ components add up. Let us denote the net electric field at point P due to the two strips as d**E** and d**E** is given by

$$d\mathbf{E} = 2\, E_{sx}\, \hat{\mathbf{i}}$$

From figure 2.10 we can write

$$\cos\theta = \frac{E_{sx}}{E_s} = \frac{t}{r}$$

$$\Rightarrow E_{sx} = \frac{t}{r}E_s$$

$$E_{sx} = \frac{t}{\sqrt{(y')^2 + t^2}}E_s$$

Substituting the value of $E_{sx}$ in d**E**

$$d\mathbf{E} = 2\frac{t}{\sqrt{(y')^2 + t^2}}E_s\, \hat{\mathbf{i}}$$

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Substituting the value of $E_s$ in d**E**

$$d\mathbf{E} = 2\frac{t}{\sqrt{(y')^2 + t^2}}\left(\frac{1}{4\pi\varepsilon_o}\right)\frac{\lambda dy'}{\left[(y')^2 + t^2\right]^2}\hat{\mathbf{i}}$$

$$d\mathbf{E} = \frac{1}{2\pi\varepsilon_o}\frac{\lambda t\, dy'}{\left[(y')^2 + t^2\right]^{3/2}}\hat{\mathbf{i}}$$

This is the field produced by two line elements $\lambda dy'$ at point P. The field produced by the entire line charge at point P is

$$\mathbf{E} = \int_0^a d\mathbf{E} = \frac{\lambda t}{2\pi\varepsilon_o}\int_0^a \frac{dy'}{\left[(y')^2 + t^2\right]^{3/2}}\hat{\mathbf{i}}$$

$$= \frac{\lambda t}{4\pi\varepsilon_o}\left[\frac{y'}{t^2\sqrt{(y')^2 + t^2}}\right]_0^a \hat{\mathbf{i}}$$

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_o}\left[\frac{a}{t\sqrt{a^2 + t^2}}\right]\hat{\mathbf{i}}$$

**Example 2.6**

Two point charges $q_1$=10 nanocoulombs and $q_2$=20 nanocoulombs are located at (1, 1, 3) m and at (2, 3, 2) m respectively. Calculate the magnitude of force on $q_1$ by $q_2$.

***Solution:***

The distance between the two charges is

$$r = \sqrt{(2-1)^2 + (3-1)^2 + (2-3)^3} = \sqrt{6}\,\text{m}$$

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The force exerted by $q_2$ on $q_1$ is given by

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o} \frac{(10\times10^{-9})(20\times10^{-9})}{(\sqrt{6})^2} \hat{\boldsymbol{r}}$$

Here $\hat{\boldsymbol{r}}$ is the unit vector pointing from $q_2$ to $q_1$

$$\mathbf{F} = 300\times10^{-9}\ \hat{\boldsymbol{r}} \text{ Newtons}$$

In order to find $\hat{\boldsymbol{r}}$ let us calculate the vector $\boldsymbol{r}$ pointing from $q_2$ to $q_1$

$$\boldsymbol{r} = (2-1)\hat{\mathbf{i}} + (3-1)\hat{\mathbf{j}} + (2-3)\hat{\mathbf{k}}$$

$$\boldsymbol{r} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

and $|\boldsymbol{r}| = \sqrt{1+4+1} = \sqrt{6}$

$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|} = \frac{\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{6}}$$

$$\mathbf{F} = \frac{300}{\sqrt{6}}\times10^{-9}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \text{ Newtons}$$

Now $|\mathbf{F}| = \frac{300\times10^{-9}}{\sqrt{6}}\sqrt{6} = 300\times10^{-9}$ Newtons

**Example 2.7**

Charge is uniformly distributed on a circular disc of radius R. Calculate the electric field at point P at distance *y* from the center of the disc. Charge density σ on the surface of the disc is uniform.



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***Solution:***

In order to calculate the electric field produced by the entire surface charge $\sigma$ divide the surface of the entire disc into number of small surface elements $ds'$. Let us consider two surface elements $ds'$ each holding a charge $\sigma ds'$ as shown in figure 2.11 a,b. Let us calculate the electric field due to these two surface elements $ds'$ initially and, finally integrate between 0 to $\pi$, 0 to R in order to calculate net electric field produced by the entire disc.

The electric field produced by the single surface element $ds'$ at point P is

$$|\mathbf{E_e}| = \frac{1}{4\pi\varepsilon_o}\frac{\sigma ds'}{\left[y^2 + (r')^2\right]}$$

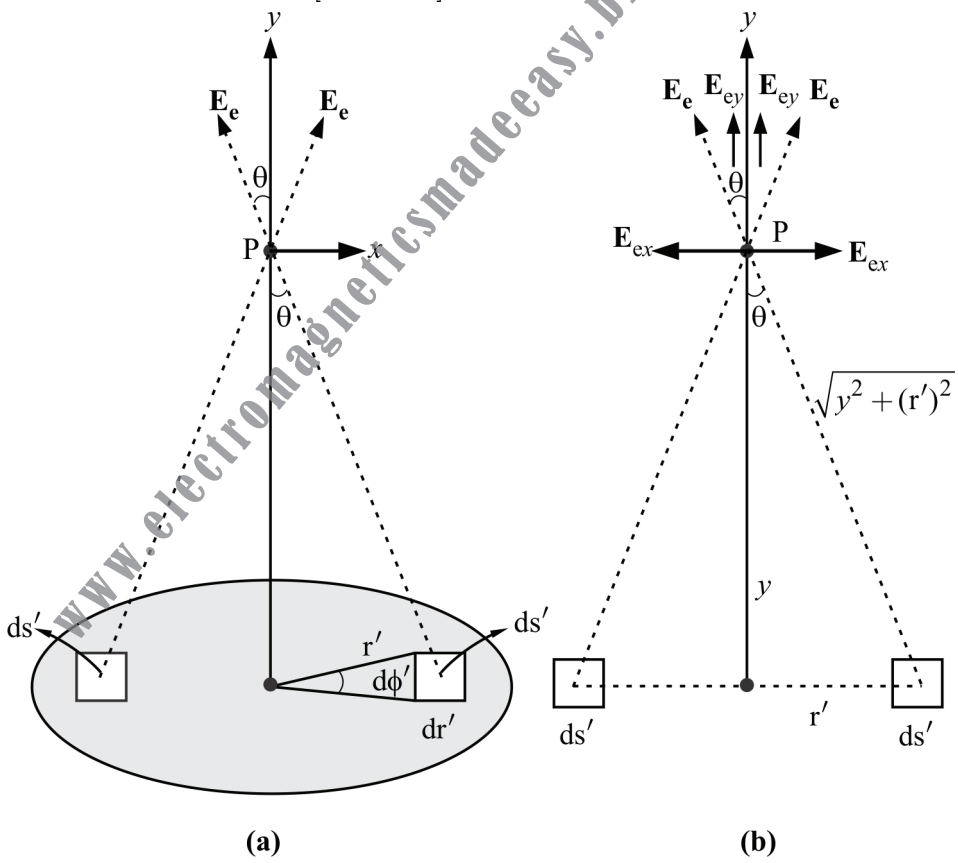


**Fig.2.11**

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Resolve $|\mathbf{E_e}|$ into $\mathbf{E}_{ex}$ and $\mathbf{E}_{ey}$ components as shown in figure 2.11b. Clearly $\mathbf{E}_{ex}$ components cancel out while $\mathbf{E}_{ey}$ components add up. Let d$\mathbf{E}$ be the electric field produced by the two elements at point P. Then as per the above discussion

$$d\mathbf{E} = 2\, E_{ey}\, \hat{\mathbf{j}}$$

From figure 2.11 b

$$\cos\theta = \frac{E_{ey}}{E_e} = \frac{y}{\sqrt{y^2 + (r')^2}}$$

$$\Rightarrow E_{ey} = \frac{y}{\sqrt{y^2 + (r')^2}} E_e$$

Substituting the value of $E_{ey}$ in d$\mathbf{E}$

$$d\mathbf{E} = 2\frac{y}{\sqrt{y^2 + (r')^2}} E_e$$

Substituting for $E_e$ in d$\mathbf{E}$

$$d\mathbf{E} = 2\frac{y}{\sqrt{y^2 + (r')^2}} \frac{1}{4\pi\varepsilon_o} \frac{\sigma ds'}{\left[y^2 + (r')^2\right]} \hat{\mathbf{j}}$$

$$d\mathbf{E} = \frac{\sigma y}{2\pi\varepsilon_o} \frac{ds'}{\left[y^2 + (r')\right]^{3/2}} \hat{\mathbf{j}}$$

Now the field produced by the entire disc is then

$$\mathbf{E} = \int d\mathbf{E}$$

$$= \frac{\sigma y}{2\pi\varepsilon_o} \int \frac{ds'}{\left[y^2 + (r')^2\right]^{3/2}} \hat{\mathbf{j}}$$

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$$\mathbf{E} = \frac{\sigma y}{2\pi\varepsilon_o} \int_0^{\pi} \int_0^{R} \frac{r' dr' d\phi'}{\left[y^2 + (r')^2\right]^{3/2}} \hat{\mathbf{j}}$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_o} \left[1 - \frac{y}{\left[y^2 + R^2\right]}\right] \hat{\mathbf{j}}$$

**Example 2.8**

A 1 ncoulomb charge is situated at (2, 0, 0) m and 2 ncoulomb charge is situated at (0, 1, 0) m. Find the electric field at point P whose coordinates are (0, 0, 4)m.

***Solution:***

The position vector of 1 ncoulomb charge with respect point P is

$$\boldsymbol{r}_1 = -2\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$$

The position vector of 2 ncoulomb charge with respect to point P is

$$\boldsymbol{r}_2 = -1\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

The electric field of 1 ncoulomb charge at point P is

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_o} \frac{1\times10^{-9}}{r_1^2} \hat{\boldsymbol{r}}_1$$

Now $|\boldsymbol{r}_1| = \sqrt{4+16} = \sqrt{20}$

$$\hat{\boldsymbol{r}}_1 = \frac{\boldsymbol{r}_1}{|\boldsymbol{r}_1|} = \frac{-2\hat{\mathbf{i}} + 4\hat{\mathbf{k}}}{\sqrt{20}}$$

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Hence, $\mathbf{E_1} = \frac{1}{4\pi\varepsilon_o} \frac{1\times10^{-9}}{20} \frac{(-2\hat{\mathbf{i}} + 4\hat{\mathbf{k}})}{\sqrt{20}}$

$$\Rightarrow \mathbf{E_1} = -0.20125\hat{\mathbf{i}} + 0.4025\hat{\mathbf{k}}$$

Now $|\boldsymbol{r_2}| = \sqrt{1+16} = \sqrt{17}$

$$\hat{\boldsymbol{r}}_2 = \frac{\boldsymbol{r_2}}{|\boldsymbol{r_2}|} = \frac{-\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{\sqrt{17}}$$

$$\Rightarrow \mathbf{E_2} = (9\times10^9) \frac{(2\times10^{-9})}{(17)} \frac{(-\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{17}}$$

$$\mathbf{E_2} = -0.2568\hat{\mathbf{j}} + 1.027\hat{\mathbf{k}}$$

The net electric field at point P is

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} = (-0.20125\mathbf{i} + 0.4025\hat{\mathbf{k}}) + (-0.2568\hat{\mathbf{j}} + 1.027\hat{\mathbf{k}})$$

$$\mathbf{E} = -0.20125\hat{\mathbf{i}} - 0.2568\hat{\mathbf{j}} + 1.4295\hat{\mathbf{k}}$$

**Example 2.9**

Three charges of magnitude $q_1$, $q_2$, $q_3$ are situated at the three corners of a square of side b. Calculate the net electric field produced at the fourth corner of the square.

***Solution:***

Consider figure 2.12. As shown the electric field produced by $q_1$ is acting along *x* axis and is given by

$$\mathbf{E_1} = |\mathbf{E_1}|\,\hat{\mathbf{i}}$$

The electric field produced by $q_3$ is acting along *y* axis and is given by

$$\mathbf{E_3} = |\mathbf{E_3}|\,\hat{\mathbf{j}}$$



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The electric field produced by $q_2$ is along the diagonal and is denoted as $\mathbf{E_2}$. The net electric field at point O is given by

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3}$$

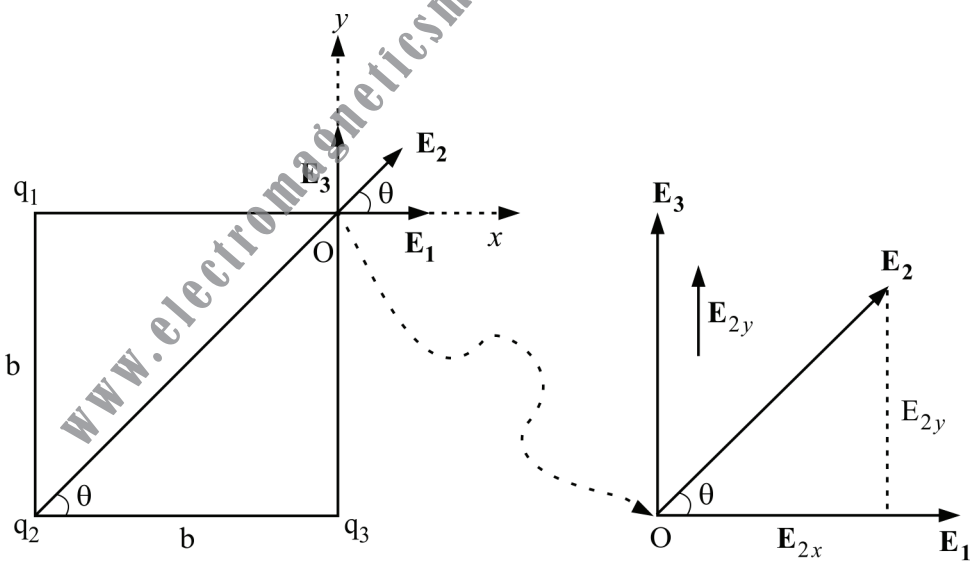
To find $\mathbf{E}$ we need to resolve $\mathbf{E_2}$ into its $x$ and $y$ components as shown in figure 2.12b

$$\mathbf{E_2} = E_{2x}\,\hat{\mathbf{i}} + E_{2y}\,\hat{\mathbf{j}}$$

Thus

$$\mathbf{E} = E_1\,\hat{\mathbf{i}} + E_{2x}\,\hat{\mathbf{i}} + E_{2y}\,\hat{\mathbf{j}} + E_3\,\hat{\mathbf{j}}$$

$$\mathbf{E} = (E_1 + E_{2x})\,\hat{\mathbf{i}} + (E_{2y} + E_3)\,\hat{\mathbf{j}}$$




**(a)** **(b)**

**Fig.2.12**

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**Example 2.10**

Three charges shown in figures (2.13) are in equilibrium. Find the value of a.

***Solution:***

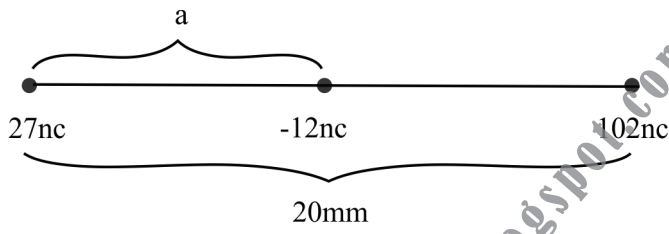


**Fig.2.13**

For the charges to be in equilibrium

$$\frac{(27)(12)}{a^2} = \frac{(27)(102)}{20} = \frac{(12)(102)}{(20-a)^2}$$

$$\Rightarrow a^2 = \frac{(27)(12)(20)}{(27)(102)}$$

$$\Rightarrow a = 1.5339 \text{ mm}$$

## 2.7 Solid angle

In the lower classes the reader would have learnt about ordinary angle which is angle in two dimensions. The ordinary angle θ is shown in figure 2.14a. It is the angle between two lines. In figure 2.14a it is the angle between line OA and OB.

When θ is in radians

$$\theta = \frac{l}{r}$$

where *l* is the arc length and OA = OB = r in figure 2.14a. As we increase OA, OB in figure 2.14b to $OA_1$, $OB_1$, θ remains the same.



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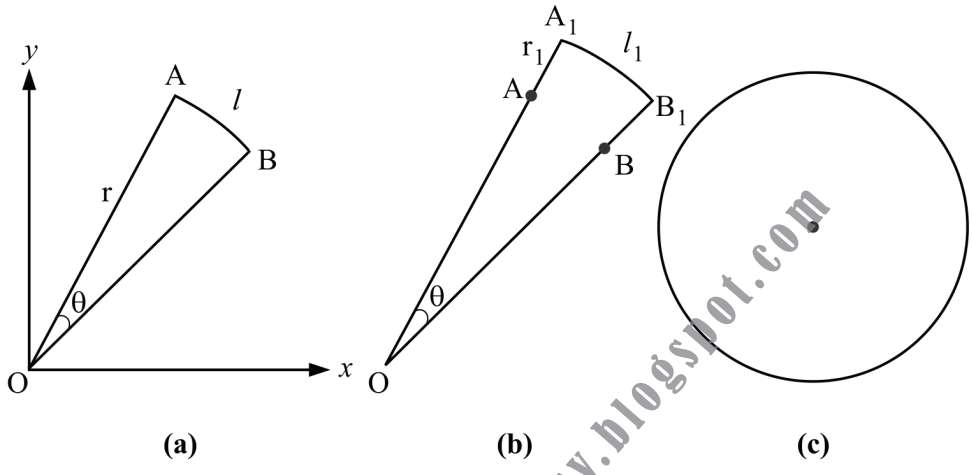


**Fig.2.14**

That is $\theta = \frac{l}{r} = \frac{l_1}{r_1}\ [OA_1 = OB_1 = r_1]$

As we increase $l$ it finally forms a circle as shown in figure 2.14c. For a circle $l = 2\pi r$.

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$

Similarly for a semicircle $l = \pi r$

$$\theta = \frac{l}{r} = \frac{\pi r}{r} = \pi$$

Thus the ordinary angle is the angle in two dimensions.

Solid angle is the angle in three dimensions. Take a plain sheet of paper and rotate it in the form of cone as shown in figure 2.15. The angle subtended is the solid angle.

The solid angle is denoted by Ω. Now consider figure 2.16a. We have shown solid circle Ω along with *x, y, z* axis. Now in figure 2.16a AB is area. The solid angle is given by

$$\Omega = \frac{\text{Area AB}}{r^2}$$



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## Electrostatics

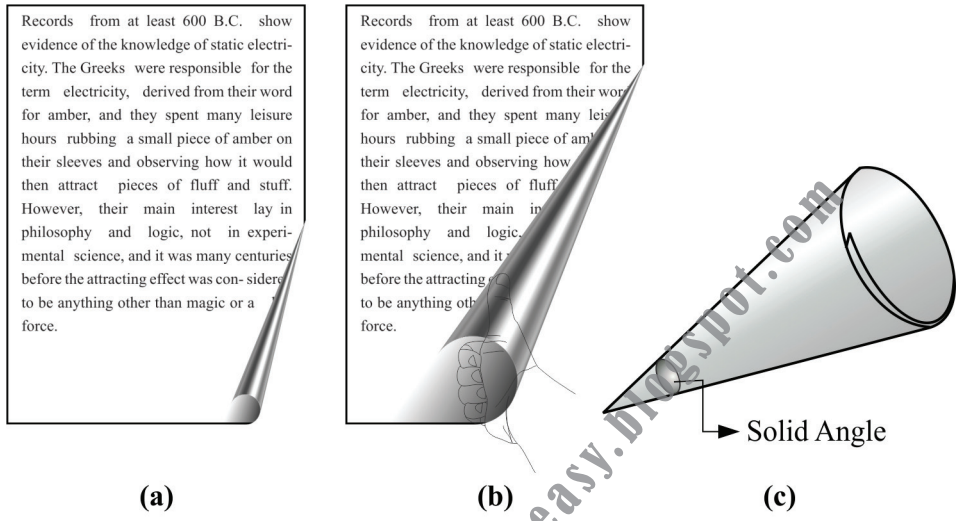


**Fig.2.15**

From figure 2.16a as r is increased to $r_1$ in figure 2.16b still Ω remains constant

$$\Omega = \frac{\text{Area AB}}{r^2} = \frac{\text{Area } A_1 B_1}{r_1^2} \left[OA_1 = OB_1 = r_1\right]$$

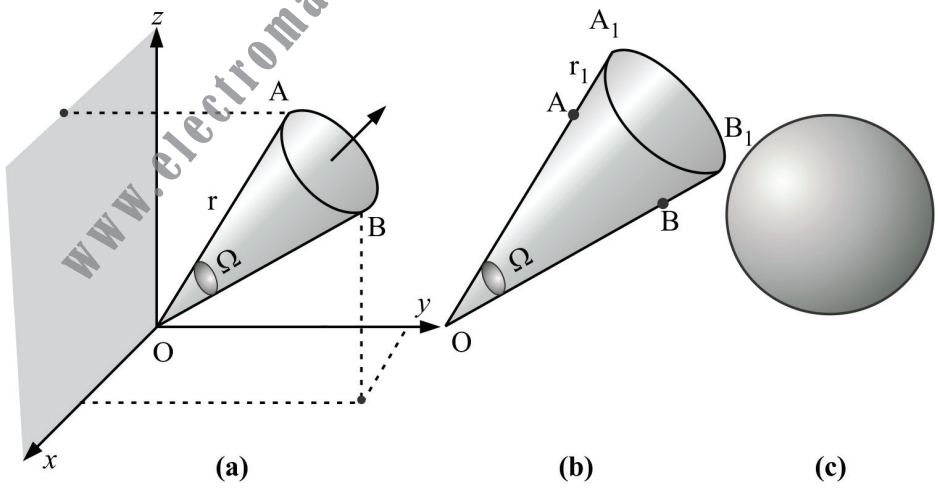


**Fig.2.16**



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As the area AB is increased and closed finally, we get a sphere as shown in figure 2.16c. For a sphere the surface area is $4\pi r^2$. Hence for a sphere solid angle is

$$\Omega = \frac{\text{Area AB}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

For a hemisphere surface area is $2\pi r^2$. Thus solid angle for hemisphere is

$$\Omega = \frac{2\pi r^2}{r^2} = 2\pi$$

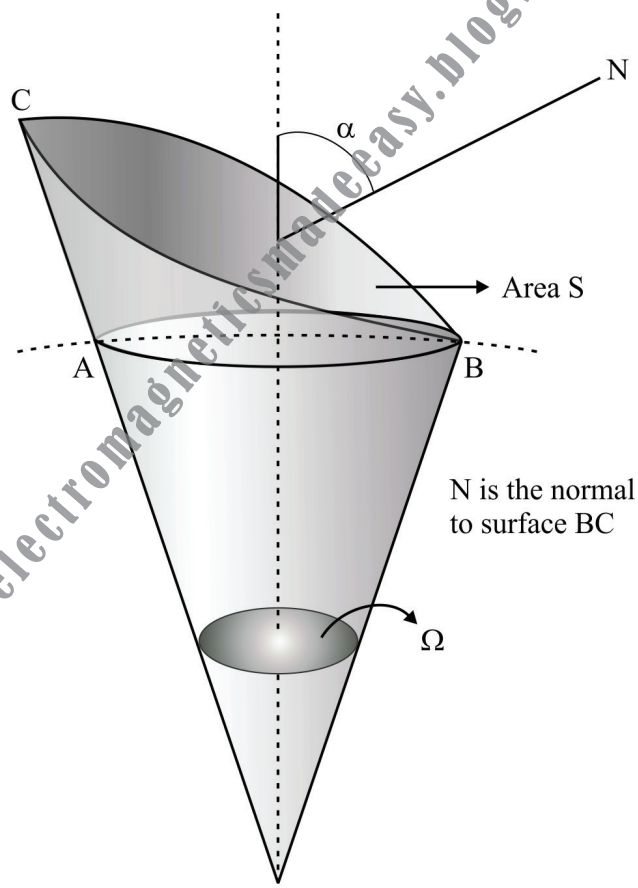


**Fig.2.17**

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For a sphere we have seen that the solid angle is $4\pi$. Although we have derived for the case of sphere the result holds good for any closed surface.

"That is, the solid angle substended by any closed surface is $4\pi$."

Now consider figure 2.17. Let area BC be S. From figure 2.17 it is clear that AB is the projection of BC. Hence from section 1.12 we can write

$$\text{Area AB} = \text{Area BC}\cos\alpha \qquad ...(2.23)$$

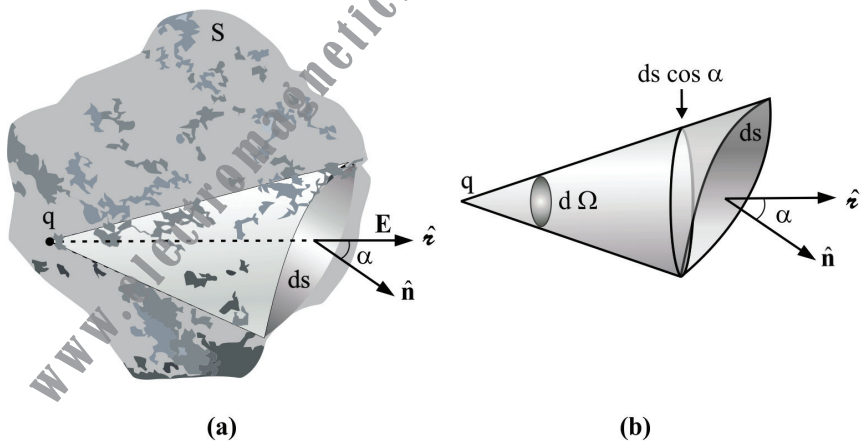
$$\Rightarrow \text{Area AB} = S\cos\alpha \qquad ...(2.24)$$

Substituting 2.24 in solid angle $\Omega$

$$\Omega = \frac{\text{Area AB}}{r^2} = \frac{S\cos\alpha}{r^2} \qquad ...(2.25)$$

## 2.8 Gauss's Law

In this section we will prove Gauss's law and later we will see how Gauss's law simplifies the calculation of electric field as compared to Coulomb's law.




**(a)** **(b)**

**Fig.2.18**

Consider an arbitrary surface S which is enclosing a charge q as shown in figure 2.18a. Let **E** be the electric field produced by q as shown in figure 2.18a. $\hat{r}$ is the unit vector in the direction of **E**. Consider a small strip of area ds and $\hat{n}$ is the unit vector normal to ds. The angle between $\hat{n}$ and $\hat{r}$ is $\alpha$.



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The flux passing through the small surface ds is

$$= \mathbf{E} \cdot d\mathbf{s} \qquad ...(2.26)$$

The total flux passing through the closed surface S is

$$= \oint_S \mathbf{E}.d\mathbf{s} \qquad ...(2.27)$$

[Reader should note that we are using unprimed symbols instead of primed symbols because we are dealing with field points].

But $\mathbf{E} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\boldsymbol{r}}$, $d\mathbf{s} = ds\,\hat{\mathbf{n}}$

Substituting the above equations in equation 2.27

$$\oint \mathbf{E}.d\mathbf{s} = \oint \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\hat{\boldsymbol{r}}.\hat{\mathbf{n}}\,ds$$

Because $\hat{\boldsymbol{r}}, \hat{\mathbf{n}}$ are unit vectors the dot product $\hat{\boldsymbol{r}}.\hat{\mathbf{n}} = \cos\alpha$. Thus

$$\text{Total flux} = \frac{q}{4\pi\varepsilon_o}\oint \frac{ds\cos\alpha}{r^2} \qquad ...(2.28)$$

But from figure 2.18b and using equation 2.25

$$\text{Total flux} = \frac{q}{4\pi\varepsilon_o}\oint d\Omega \qquad ...(2.29)$$

where $d\Omega$ is the solid angle as shown in figure 2.18b. But $\oint d\Omega = 4\pi$ because the solid angle subtended by any closed surface is $4\pi$. Hence,

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q}{4\pi\varepsilon_o}4\pi = \frac{q}{\varepsilon_o} \qquad ...(2.30)$$

Suppose the charge q is situated outside the surface S as shown in figure 2.19. Consider two small strips $ds_1$ and $ds_2$. At surface $ds_1$ the electric field is

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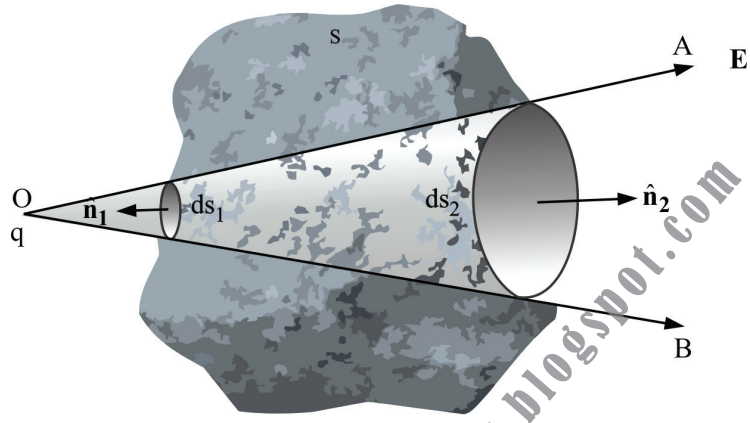


**Fig.2.19**

pointing opposite to $\hat{\mathbf{n}}_1$, the unit vector normal to $ds_1$. Hence flux at $ds_1$ is [using equation 2.30]

$$= \frac{-q d\Omega}{4\pi\varepsilon_o}$$

[Here we need not integrate because we are considering only small strip $ds_1$ and not the entire surface S].

At $ds_2$ the **E** is pointing in the same direction of $\hat{\mathbf{n}}_2$ hence the flux at $ds_2$ is

$$= \frac{q d\Omega}{4\pi\varepsilon_o}$$

Hence the net flux through the elements $ds_1$ and $ds_2$ is

$$= -\frac{q d\Omega}{4\pi\varepsilon_o} + \frac{q d\Omega}{4\pi\varepsilon_o} = 0$$

The above fact is true for any cone drawn through the surface S in figure 2.19. Thus if the charge is lying outside the surface the net flux enclosed by the surface due to the charge is zero.



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In figure 2.18 if the charge is situated in a medium of dielectric constant $\varepsilon_1$ then the flux is

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q}{\varepsilon_o \varepsilon_r} \qquad ...(2.31)$$

In figure 2.18 instead of one charge if the surface S encloses n number of charges $q_1$, $q_2$, $q_3$.....$q_n$ then told flux enclosed by the surface S is

$$\text{Total flux} = (\text{flux})_1 + (\text{flux})_2 + ..... + (\text{flux})_n \qquad ...(2.32)$$

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + ..... + \frac{q_n}{\varepsilon_o} \qquad ...(2.33)$$

Here **E** is the "net" electric field due to all charges

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_1 + q_2 + q_3 + ..... + q_n}{\varepsilon_o} \qquad ...(2.34)$$

$$\oint_S \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.35)$$

where $q_1 + q_2 + ..... + q_n = q_{enc}$ is the total charge enclosed by the surface S and $(Flux)_1$, $(Flux)_2$, ....$(Flux)_n$ are the fluxes through the surface S due to charges $q_1$, $q_2$ .....$q_n$.

From equation 2.22

$$q_{enc} = \oint_\tau \rho \, d\tau \qquad ...(2.36)$$

assuming that a volume charge densits ρ is enclosed in surface S in figure 2.18. τ is the volume enclosed by surface S in figure 2.18. In equation 2.36 we use unprimed symbol instead of primed symbol because we are dealing with field points.

With aid of Gauss divergence theorem

$$\oint_S \mathbf{E}.d\mathbf{s} = \int_\tau \nabla.\mathbf{E} \, d\tau \qquad ...(2.37)$$

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Substituting equations 2.37, 2.36 in equation 2.35

$$\int_{\tau} \nabla . \mathbf{E}\, d\tau = \frac{\int_{\tau} \rho\, d\tau}{\varepsilon_o} \qquad ...(2.38)$$

The above integration considers a small volume $d\tau$ and then integrates over the entire volume $\tau$. Equation 2.38 is valid even for the small volume $d\tau$. In that case we need not integrate. Hence,

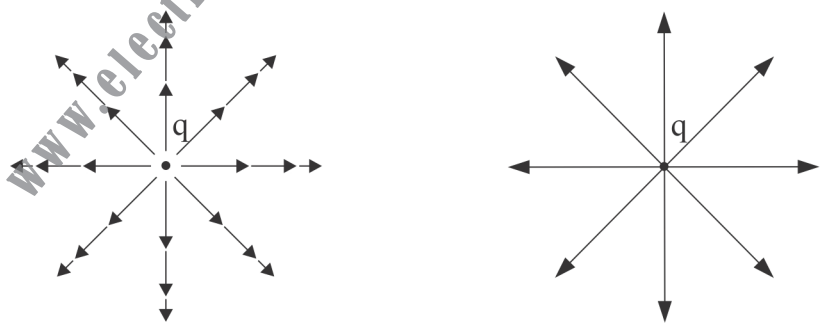
$$\nabla . \mathbf{E}\, d\tau = \frac{\rho\, d\tau}{\varepsilon_o} \qquad ...(2.39)$$

$$\Rightarrow \nabla . \mathbf{E} = \frac{\rho}{\varepsilon_o} \qquad ...(2.40)$$

The above form of Gauss's law is called differential or point form of Gauss's law.

## 2.9 Sketches of Field Lines

Equation 2.40 tells us the fact divergence of the electric field produced by a static charge distribution is non zero. Recall that divergence means spreading out. Equation 2.40 says that the electric field of a static charge distribution spreads out. For example we sketch the electric field of a point charge in figure 2.20a and see that electric field spreads out.




**(a)** **(b)**

**Fig.2.20**

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We have sketched the electric field in figure 2.20a in terms of vectors. However instead of sketching the field in terms of vectors as in figure 2.20a we can sketch the field in terms of "field lines" as shown in figure 2.20b.

Few important points to be noted in the case of field lines are

a) Field lines do not cross each other, otherwise electric field would have two different directions at that point.

b) Field lines start on positive charge and end on negative charge as shown in figure 2.21a otherwise, they extend to infinity as shown in figure 2.21b.

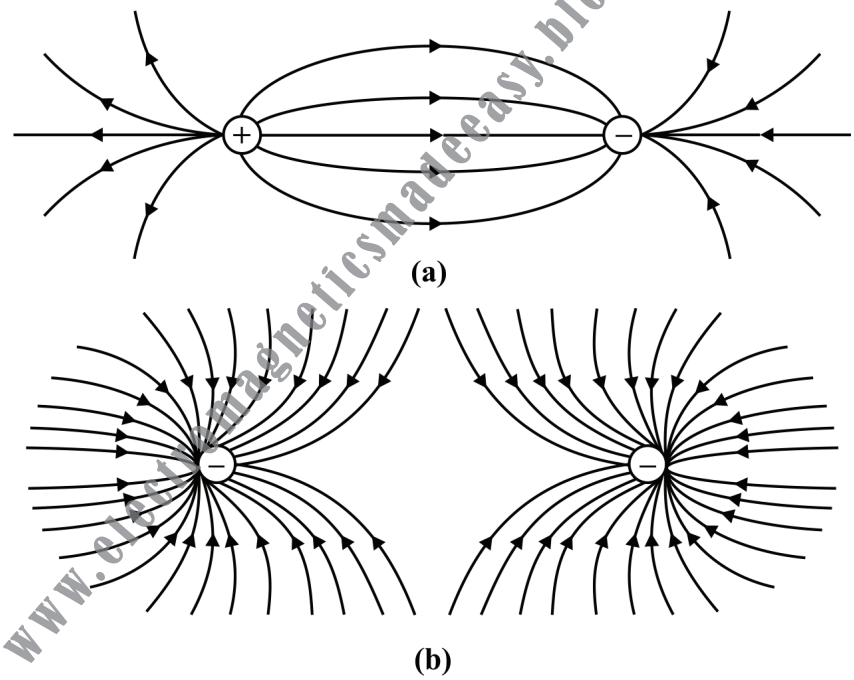


**Fig.2.21**

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One of the examples in which we used field lines is in defining flux in section 1.12 and utilising it in deriving Gauss law in section 2.8.



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## 2.10 Curl of E

Consider a point charge q situated at origin as shown in figure 2.22. The electric field at point P is given by

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_o}\frac{1}{r^2}\frac{\mathbf{r}}{|\mathbf{r}|} \qquad ...(2.41)$$

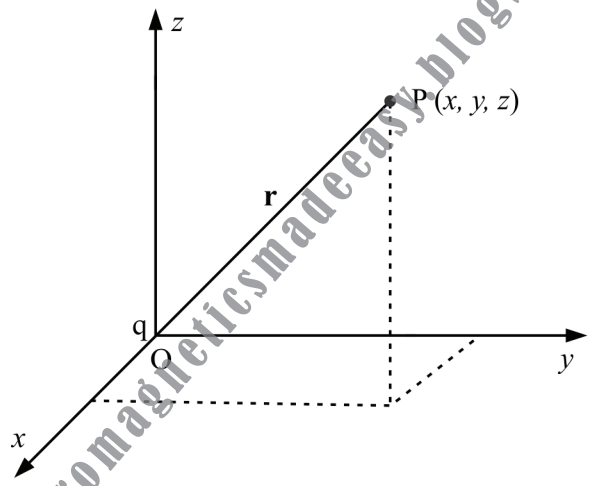


**Fig.2.22**

From figure 2.22

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \qquad ...(2.42)$$

$$|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2} \qquad ...(2.43)$$

Substituting equation 2.43, 2.42 in 2.41

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_o}\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}}{\left[x^2 + y^2 + z^2\right]^{3/2}} \qquad ...(2.44)$$

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$$\Rightarrow \mathbf{E} = \frac{q}{4\pi\varepsilon_o} \frac{x}{\left[x^2 + y^2 + z^2\right]^{3/2}} \hat{\mathbf{i}}$$

$$+ \frac{q}{4\pi\varepsilon_o} \frac{y}{\left[x^2 + y^2 + z^2\right]^{3/2}} \hat{\mathbf{j}}$$

$$+ \frac{q}{4\pi\varepsilon_o} \frac{z}{\left[x^2 + y^2 + z^2\right]^{3/2}} \hat{\mathbf{k}} \qquad ...(2.45)$$

$$\Rightarrow \mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}} \qquad ...(2.46)$$

where

$$E_x = \frac{q}{4\pi\varepsilon_o} \frac{x}{\left[x^2 + y^2 + z^2\right]^{3/2}} \qquad ...(2.47)$$

$$E_y = \frac{q}{4\pi\varepsilon_o} \frac{y}{\left[x^2 + y^2 + z^2\right]^{3/2}} \qquad ...(2.48)$$

$$E_z = \frac{q}{4\pi\varepsilon_o} \frac{z}{\left[x^2 + y^2 + z^2\right]^{3/2}} \qquad ...(2.49)$$

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We know that

$$\nabla \times \mathbf{E} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathrm{E}_x & \mathrm{E}_y & \mathrm{E}_z \end{vmatrix}$$

$$= \left( \frac{\partial \mathrm{E}_z}{\partial y} - \frac{\partial \mathrm{E}_y}{\partial z} \right) \hat{\mathbf{i}} + \left( \frac{\partial \mathrm{E}_x}{\partial z} - \frac{\partial \mathrm{E}_z}{\partial x} \right) \hat{\mathbf{j}} + \left( \frac{\partial \mathrm{E}_y}{\partial x} - \frac{\partial \mathrm{E}_x}{\partial y} \right) \hat{\mathbf{k}} \quad \ldots(2.50)$$

Let us calculate

$$\left( \frac{\partial \mathrm{E}_z}{\partial y} - \frac{\partial \mathrm{E}_y}{\partial z} \right) \quad \ldots(2.51)$$

$$\frac{\partial \mathrm{E}_z}{\partial y} = \frac{\mathrm{q}}{4\pi\varepsilon_\mathrm{o}} \frac{\partial}{\partial y} \left[ \frac{z}{\left( x^2 + y^2 + z^2 \right)^{3/2}} \right]$$

$$\frac{\partial \mathrm{E}_z}{\partial y} = \frac{-3\mathrm{q}}{4\pi\varepsilon_\mathrm{o}} \left[ \frac{z\,y}{\left( x^2 + y^2 + z^2 \right)^{5/2}} \right] \quad \ldots(2.52)$$

$$\frac{\partial \mathrm{E}_y}{\partial z} = \frac{\mathrm{q}}{4\pi\varepsilon_\mathrm{o}} \frac{\partial}{\partial z} \left[ \frac{y}{\left( x^2 + y^2 + z^2 \right)^{3/2}} \right]$$

$$= \frac{-3\mathrm{q}}{4\pi\varepsilon_\mathrm{o}} \left[ \frac{y\,z}{\left( x^2 + y^2 + z^2 \right)^{5/2}} \right] \quad \ldots(2.53)$$

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Now

$$\left(\frac{\partial \mathrm{E}_z}{\partial y}-\frac{\partial \mathrm{E}_y}{\partial z}\right)$$

$$=\frac{-3\mathrm{q}}{4\pi\varepsilon_\mathrm{o}}\left[\frac{z\,y}{\left(x^2+y^2+z^2\right)^{5/2}}\right]+\frac{3\mathrm{q}}{4\pi\varepsilon_\mathrm{o}}\left[\frac{y\,z}{\left(x^2+y^2+z^2\right)^{5/2}}\right]$$

$$\Rightarrow\left(\frac{\partial \mathrm{E}_z}{\partial y}-\frac{\partial \mathrm{E}_y}{\partial z}\right)=0 \qquad ...(2.54)$$

Similarly

$$\frac{\partial \mathrm{E}_x}{\partial z}-\frac{\partial \mathrm{E}_z}{\partial x}=0 \qquad ...(2.55)$$

$$\frac{\partial \mathrm{E}_y}{\partial x}-\frac{\partial \mathrm{E}_x}{\partial y}=0 \qquad ...(2.56)$$

Substituting equations 2.54, 2.55, 2.56 in equation 2.50

$$\nabla\times\mathbf{E}=0 \qquad ...(2.57)$$

Thus curl of **E** is zero. Suppose there are number of charges each producing the field $\mathbf{E_1}, \mathbf{E_3}, .....\mathbf{E_n}$ then from equation 2.11

$$\mathbf{E}=\mathbf{E_1}+\mathbf{E_2}+.....+\mathbf{E_n} \qquad ...(2.58)$$

With the help of equation 2.57 we see that

$$\left.\begin{aligned}\nabla\times\mathbf{E_1}&=0\\ \nabla\times\mathbf{E_2}&=0\\ &.....\\ \nabla\times\mathbf{E_n}&=0\end{aligned}\right\} \qquad ...(2.59)$$

for individual charges.

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Thus for the net electric field **E** due to all charges given in equation 2.58

$$\nabla \times \mathbf{E} = \nabla \times \mathbf{E_1} + \nabla \times \mathbf{E_2} + ..... + \nabla \times \mathbf{E_n} \quad ...(2.60)$$

From equation 2.59 then

$$\nabla \times \mathbf{E} = 0 \quad ...(2.61)$$

The above equation is true for any static charge distribution. Recall that curl is a measure of circulation or rotation. $\nabla \times \mathbf{E} = 0$ says that electric fields due to static charges do not circulate about. Electric fields due to static charges are irrotational fields.

## 2.11 Potential of Discrete and Continuous Charge Distributions

In this section we will derive expression for potential. In the later sections we will see how potential simplifies the calculation of electric field as compared to Coulomb's law.

From equation 2.61

$$\nabla \times \mathbf{E} = 0 \quad ...(2.62)$$

From example 1.9, curl gradient of any scalar is zero. Hence,

$$\nabla \times (-\nabla \mathrm{V}) = 0 \quad ...(2.63)$$

Here V is a scalar quantity and is known as electric potential. The inclusion of the negative sign in the above equation will be explained later.

Comparing equations 2.62 and 2.63

$$\mathbf{E} = -\nabla \mathrm{V} \quad ...(2.64)$$

$$\Rightarrow \mathrm{V} = -\int_{\mathrm{R}}^{\mathrm{P}} \mathbf{E}.\mathrm{d}\boldsymbol{l} \quad ...(2.65)$$

Where R the reference point is the "zero of the potential" i.e., where we fix the potential to be zero and P is the point where the potential V is calculated. We will learn more about point R in future sections. For the time being let us fix the "zero of the potential" at ∞.



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Hence $V = -\int_{\infty}^{P} \mathbf{E}.d\boldsymbol{l}$ ...(2.66)

Consider figure 2.23. The potential at ∞ is zero. Charge q is located at origin. The point P lies at a distance r from charge q.

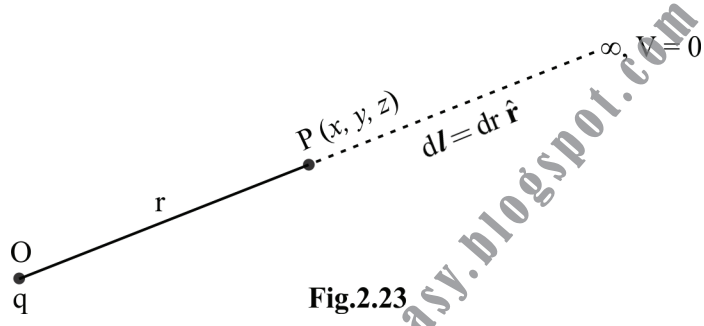


**Fig.2.23**

In figure 2.23

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\mathbf{r}} \qquad ...(2.67)$$

From figure 2.23

$$d\boldsymbol{l} = dr\,\hat{\mathbf{r}} \qquad ...(2.68)$$

From equations 2.67, 2.68

$$\mathbf{E}.d\boldsymbol{l} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r^2}\right)\hat{\mathbf{r}}.(dr\,\hat{\mathbf{r}}) \qquad ...(2.69)$$

$$\mathbf{E}.d\boldsymbol{l} = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r^2}\right)dr \qquad ...(2.70)$$

Substituting the equation 2.70 in equation 2.66

$$V = -\int_{\infty}^{r}\frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r^2}\right)dr \qquad ...(2.71)$$

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$$V = -\frac{q}{4\pi\varepsilon_o}\left[\frac{-1}{r}\right]_{\infty}^{r} \qquad ...(2.72)$$

$$V = \frac{q}{4\pi\varepsilon_o}\frac{1}{r} \qquad ...(2.73)$$

The negative sign in equation 2.63 was selected to make the potential of the positive charge come out positive.

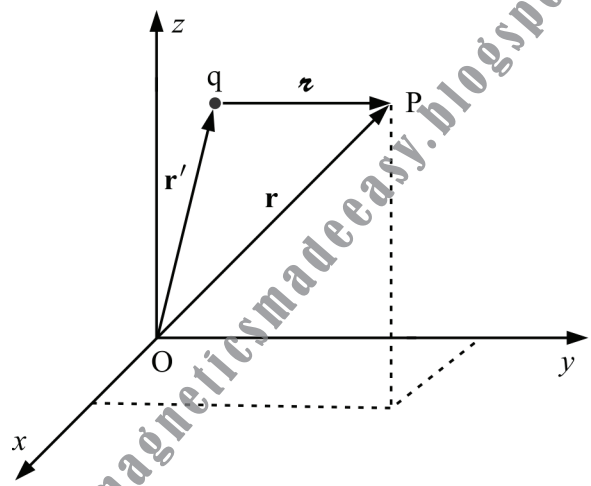


**Fig.2.24**

Equation 2.73 holds good for figure 2.23 where the point charge q lies at origin. In general the point charge q need not lie in origin as shown in figure 2.24. For such situations the potential is given by

$$V = \frac{q}{4\pi\varepsilon_o}\frac{1}{r} \qquad ...(2.74)$$

If there is a collection of n number of point charges as shown in figure 2.4 then the total potential at point P can be calculated as follows. From equation 2.11 the net electric field at point P in figure 2.4 is

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + ..... + \mathbf{E_n} \qquad ...(2.75)$$



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Integrating

$$\int_{\infty}^{P} \mathbf{E}.d\boldsymbol{l} = \int_{\infty}^{P} \mathbf{E_1}.d\,\boldsymbol{l_1} + \int_{\infty}^{P} \mathbf{E_2}.d\,\boldsymbol{l}_2 + ..... + \int_{\infty}^{P} \mathbf{E_n}.\,d\,\boldsymbol{l_n} \qquad ...(2.76)$$

$$\Rightarrow V = V_1 + V_2 + ..... + V_n \qquad ...(2.77)$$

using equation 2.74 we get

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r_2} + ..... + \frac{1}{4\pi\varepsilon_o}\frac{q_n}{r_n} \qquad ...(2.78)$$

$$V = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_n}{r_n} \qquad ...(2.79)$$

Thus when there are collection of point charges in figure 2.4 the potential at point P in figure 2.4 is given by equations 2.77, 2.78, 2.79. Here $r_1, r_2, r_3 ..... r_n$ are the distances of charges $q_1, q_2, q_3 ..... q_n$ upto point P respectively in figure 2.4.

For continuous charge distributions shown in figure 2.5 a, b, c the potential is

For line charge in figure 2.5a

$$V = \frac{1}{4\pi\varepsilon_o}\int_{l'}\frac{\lambda dl'}{r} \qquad ...(2.80)$$

For surface charge in figure 2.5b

$$V = \frac{1}{4\pi\varepsilon_o}\int_{S'}\frac{\sigma ds'}{r} \qquad ...(2.81)$$

For volume charge in figure 2.5c

$$V = \frac{1}{4\pi\varepsilon_o}\int_{\tau'}\frac{\rho\, d\tau'}{r} \qquad ...(2.82)$$

For a point charge situated at origin in figure 2.22 **E** from equation 2.46 is

$$\mathbf{E} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} + E_z\,\hat{\mathbf{k}} \qquad ...(2.83)$$

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Where the components $E_x$, $E_y$, $E_z$ are given by equations 2.47, 2.48, 2.49.

For a point charge situated in origin in figure 2.23 the potential is given by equation 2.73

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{x^2+y^2+z^2}}$$

As we see there are no components for V because V is a scalar. Vectors alone have components. Scalars don't have components. This point has been discussed in detail in section 1.9.

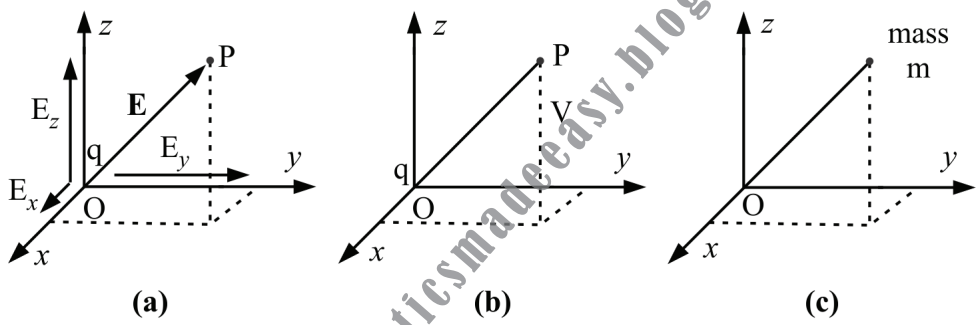


**Fig.2.25**

Electric field **E** is a vector and hence can be resolved into components as shown in figure 2.25a. However the potential at point P in figure 2.25b is V and does not have components. That is along *x*, *y*, *z* axis the potential is V, not $V_x$, $V_y$, $V_z$. To make this point further clear consider an object of mass m in figure 2.25c. The mass of the object along *x*, *y*, *z* axis is m, not $m_x$, $m_y$, $m_z$, because mass is a scalar quantity. If the mass of the object is 50kg it will be always 50kg along *x*, *y*, *z* axis. Similarly the potential V is a scalar and hence along *x*, *y*, *z* axis its value is V and not $V_x$, $V_y$, $V_z$. However the electric field **E** is a vector and hence it can be split up into three components $E_x$, $E_y$, $E_z$.

Hence if you are using Coulomb's law directly, to calculate electric field then you have to work with three components $E_x$, $E_y$, $E_z$. Hence you have to fuss with direction. However if you are working with potential you are working with



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only one component V alone and need not bother about direction because along *x*, *y*, *z* directions the value is V alone. This makes working with potential V, easy as compared to working with **E** directly. Once V has been found then **E** can be calculated using the relation

$$\mathbf{E} = -\nabla \mathrm{V}$$

The way in which potential simplifies calculation of electric field as compared to Coulomb's law will be more clear to the reader when the reader goes through section 2.18.

A final point. The potentials calculated in equations 2.80, 2.81, 2.82 will tend to infinity if the charge distribution itself extends to infinity. In that case we select a suitable different reference point.

## 2.12 Calculating Electric Field Using Gauss law and Potential

We know that Gauss's law is

$$\oint \mathbf{E}.\mathrm{d}\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

Our aim is to calculate **E** using the above equation. However **E** is inside the integral. If we want to calculate **E** using the above equation then **E** must be taken out of the integral. For this, for a given charge distribution we construct a imaginary Gaussian surface such that the following conditions are satisfied.

1) On the Gaussian surface $\mathbf{E}.\mathrm{d}\mathbf{s} = \mathrm{E}\,\mathrm{ds}\cos\theta$ can be easily evaluated.

2) On the Gaussian surface E cos θ is constant so that in the integral $\oint \mathbf{E}.\mathrm{d}\mathbf{s}$, E cos θ can be pulled out.

That is

$$\oint \mathbf{E}.\mathrm{d}\mathbf{s} = \oint \mathrm{E}\cos\theta\ \mathrm{ds} \qquad ...(2.84)$$

Because E cos θ is constant over Gaussian surface

$$\oint \mathrm{E}\cos\theta\,\mathrm{ds} = \mathrm{E}\cos\theta \oint \mathrm{ds} \qquad ...(2.85)$$



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Now $\oint ds$, the calculation of area of the Gaussian surface is simple mathematics. Thus from Gauss law

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

$$\Rightarrow \oint E\,ds\,\cos\theta = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.86)$$

Because E cos θ is constant over Gaussian surface

$$E\cos\theta \oint ds = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.87)$$

$$\Rightarrow E = \frac{q_{enc}}{\varepsilon_o}\,\frac{1}{\cos\theta \oint ds} \qquad ...(2.88)$$

Hence from the above equation E can be easily calculated.

If for a given charge distribution, a Gaussian surface cannot be constructed such that **E**.d**s** cannot be easily evaluated (or) if E cos θ is not constant over Gaussian surface, then **E** cannot be pulled out of the integral. In such situations calculation of **E** using Gauss's law becomes difficult.

Then the alternative method to calculate **E** is by calculating potential initally and then determining **E** using the relation

$$\mathbf{E} = -\nabla V$$

In the following sections we will calculate the electric field by using Coulomb's law, Gauss's law and potential for different charge distributions and compare the results and methods.

## 2.13 Electric Field Due to an Infinite Line of Charge

Consider an infinite uniform line charge present along *y* axis as shown in figure 2.26. The charge per unit length of the line charge is λ. Let us calculate the electric field at pont P which is situated at a distance of r from point O as shown in



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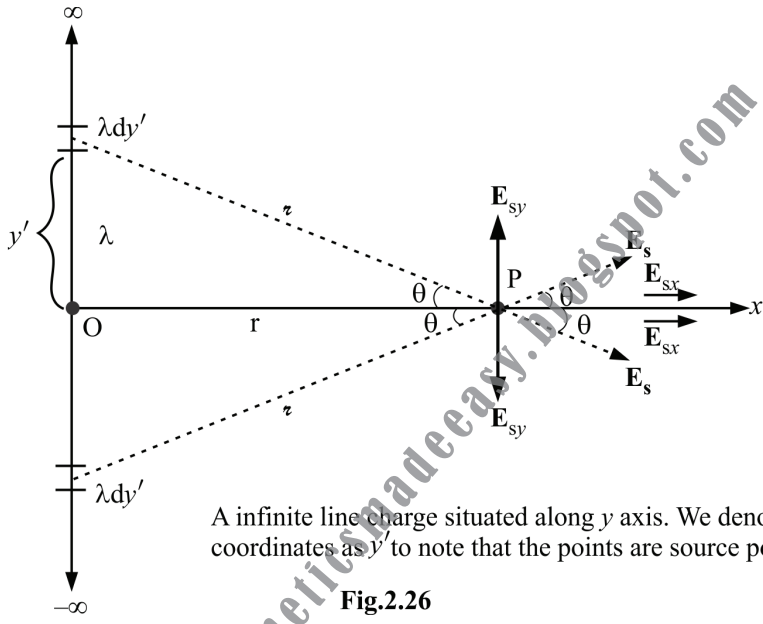


A infinite line charge situated along $y$ axis. We denote the coordinates as $y'$ to note that the points are source points.

**Fig.2.26**

figure 2.26. Now let us calculate the electric field at point P using Coulomb's law, Gauss law, potential formulation.

**a) Using Coulomb's Law**

Let us divide the entire line charge into small strips $dy'$ and consider only two strips as shown in figure 2.26. Each strip holds a charge of $\lambda\, dy'$. Let us initially calculate the electric field due to these two strips. Finally we integrate to calculate the net electric field at point P due to the entire line charge.

Resolving the electric field $\mathbf{E}_s$ due to each strip into $x$ and $y$ components $\mathbf{E}_{sx}$ and $\mathbf{E}_{sy}$ we see that, $\mathbf{E}_{sy}$ components cancel out each other while $\mathbf{E}_{sx}$ components add up. Let the net electric field at point P due to the two strips be d$\mathbf{E}$. The d$\mathbf{E}$ is given by

$$d\mathbf{E} = 2\, E_{sx}\, \hat{\mathbf{i}} \qquad ...(2.89)$$



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From figure 2.26

$$\cos\theta = \frac{E_{sx}}{E_s} \qquad ...(2.90)$$

$$E_{sx} = E_s \cos\theta \qquad ...(2.91)$$

Substituting 2.91 in 2.89

$$d\mathbf{E} = 2E_s \cos\theta\, \hat{\mathbf{i}} \qquad ...(2.92)$$

The electric field $\mathbf{E}_s$ produced by the single element is given by

$$E_s = \frac{1}{4\pi\varepsilon_o} \frac{\lambda dy'}{r^2} \qquad ...(2.93)$$

From figure 2.26

$$y' = r\tan\theta \;\Rightarrow\; dy' = r\sec^2\theta\, d\theta \qquad ...(2.94)$$

and $$r = \frac{r}{\cos\theta} \qquad ...(2.95)$$

Substituting equations 2.95, 2.94 in 2.93

$$E_s = \frac{1}{4\pi\varepsilon_o} \frac{\lambda r\sec^2\theta\, d\theta}{r^2} \cos^2\theta \qquad ...(2.96)$$

$$E_s = \frac{1}{4\pi\varepsilon_o} \frac{\lambda}{r} d\theta \qquad ...(2.97)$$

Substituting equation 2.97 in 2.92

$$d\mathbf{E} = 2.\frac{1}{4\pi\varepsilon_o} \frac{\lambda}{r} d\theta \cos\theta\, \hat{\mathbf{i}} \qquad ...(2.98)$$

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The field produced by the entire line charge at point P is

$$\mathbf{E} = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}\int_0^{\pi/2}\cos\theta\, d\theta\ \hat{\mathbf{i}} \qquad ...(2.99)$$

In the above integration we are integrating from point O to ∞, only half of the entire line charge because we are considering strips on two sides. For the point O the angle θ is zero. For the infinite point, as $y \to \infty$ the angle $\theta \to \pi/2$. Hence

$$\mathbf{E} = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}[\sin\theta]_0^{\pi/2}\hat{\mathbf{i}} \qquad ...(2.100)$$

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_o r}\hat{\mathbf{i}} \qquad ...(2.101)$$

**b) Using Gauss's law**

In figure 2.27 we show the infinite line of charge. We have to calculate field due to infinite line of charge by applying Gauss's law.

The first step is to get "the feel of the field". In figure 2.26 we see that only *x* components survive, while *y* components cancel out. Hence the field of the infinite line charge "looks" radial as shown in figure 2.27a. Next step is to construct a Gaussian surface, keeping in mind that evaluation of **E**.d**s** must be easy and E cos θ must be constant over the Gaussian surface. A little thought shows that the cylindrical Gaussian surface satisfies the above requirement. So we construct the cylindrical Gaussian surface as shown in figure 2.27b. The Gaussian surface is an imaginary surface. Let r and L be the radius and length of the cylindrical Gaussian surface. Applying Gauss's law to the cylindrical Gaussian surface

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

$$\Rightarrow \int_{\text{End face 1}} \mathbf{E}.d\mathbf{s} + \int_{\text{Curved face}} \mathbf{E}.d\mathbf{s} + \int_{\text{End face 2}} \mathbf{E}.d\mathbf{s} = \frac{\lambda L}{\varepsilon_o} \qquad ...(2.102)$$

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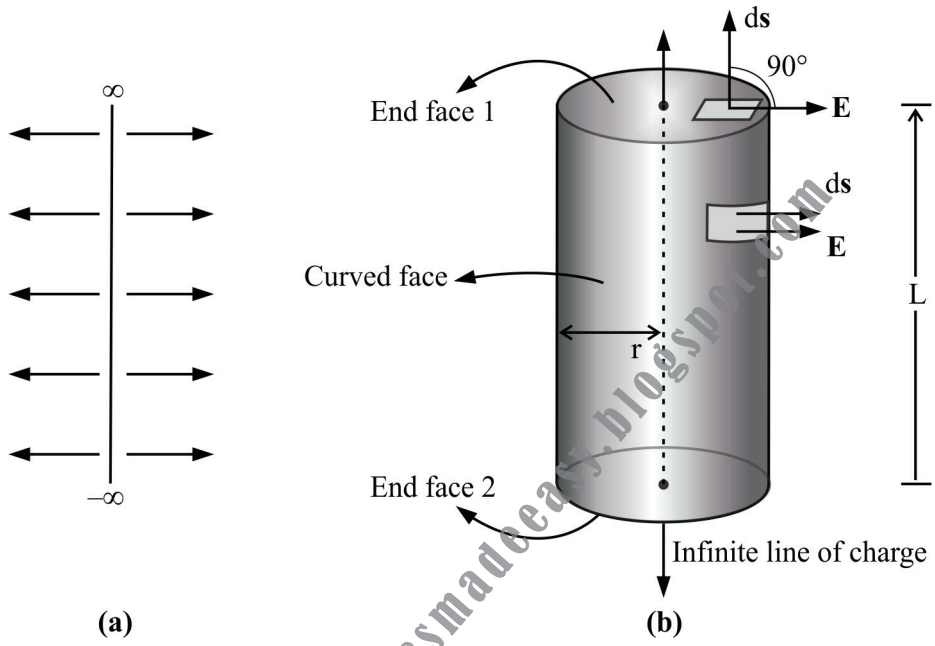


**Fig.2.27**

where $q_{enc} = \lambda L$ is the total charge enclosed by the Gaussian surface. End face 1, 2 and curved face of the cylindrical Gaussian surface are shown in figure 2.27b.

At the end faces the angle between **E** and d**s** is 90° and at the curved surface the angle between **E**, d**s** is zero degrees

Hence equation 2.102 is

$$\underset{\text{End face1}}{\int E\,ds\cos 90^\circ} + \underset{\text{Curved face}}{\int E\,ds\cos 0^\circ} + \underset{\text{End face 2}}{\int E\,ds\cos 90^\circ} = \frac{\lambda L}{\varepsilon_o} \qquad ...(2.103)$$

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As cos 90° = 0, cos 0° = 1 equation 2.103 is

$$\int_{\substack{\text{Curved}\\\text{face}}} E\, ds = \frac{\lambda L}{\varepsilon_o} \qquad ...(2.104)$$

From figure 2.27b it is clear that E is constant over the curved surface and hence

$$E \int_{\substack{\text{Curved}\\\text{face}}} ds = \frac{\lambda L}{\varepsilon_o} \qquad ...(2.105)$$

Now $\int_{\substack{\text{Curved}\\\text{face}}} ds$ is the area of the curved face which is equal to $2\pi rL$. Hence

$$\Rightarrow E(2\pi rL) = \frac{\lambda L}{\varepsilon_o} \qquad ...(2.106)$$

$$E = \frac{\lambda}{2\pi r\, \varepsilon_o} \qquad ...(2.107)$$

While calculating electric field using Coulomb's law the $E_y$ components cancelled out as shown in figure 2.26 and hence the difficulty involved in calculating electric field using Coulomb's law is less obvious in the above example. However future examples will show that Gauss's law eases out the calculation of electric field as compared to Coulomb's law.

**c) Using Potential**

Let us redraw figure 2.26 as shown in figure 2.28. Let us calculate the potential at point P in figure 2.28 due to a small element $dy'$ which carries a charge of $\lambda dy'$. We have

$$V = \frac{1}{4\pi\varepsilon_o} \frac{\lambda dy'}{r} \qquad ...(2.108)$$

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The total potential due to the entire line charge is

$$V = \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \frac{\lambda dy'}{r} \qquad ...(2.109)$$

From figure 2.28 $r^2 = (y')^2 + r^2$. Hence

$$V = \frac{1}{4\pi\varepsilon_o} \int_{-\infty}^{\infty} \frac{\lambda dy'}{\sqrt{(y')^2 + r^2}} \qquad ...(2.110)$$

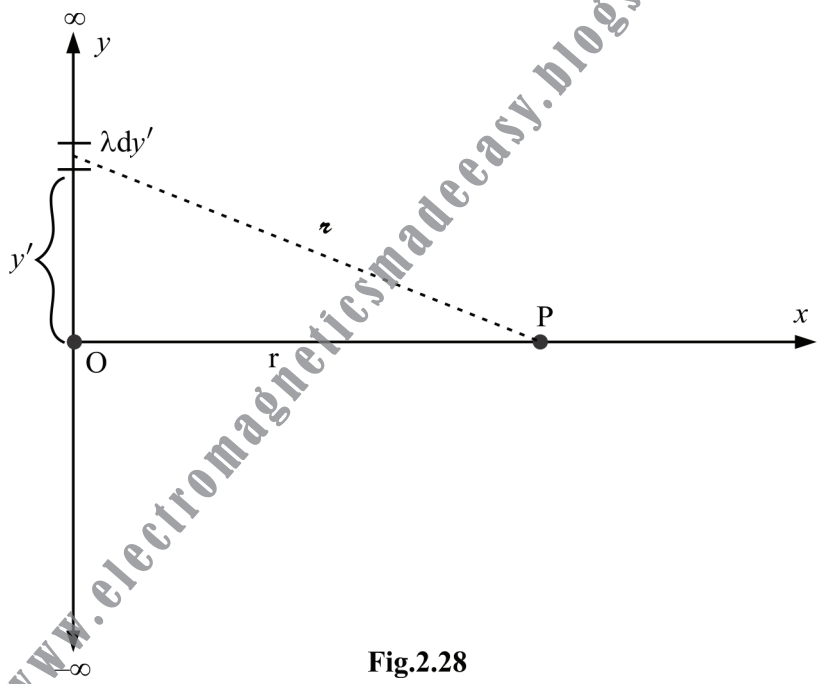


**Fig.2.28**

$$V = \frac{\lambda}{4\pi\varepsilon_o} \left[ ln\left( y' + \sqrt{(y')^2 + r^2} \right) \right]_{-\infty}^{\infty} \qquad ...(2.111)$$

$V = \infty$

As previously mentioned if the charge distribution extends to infinity then the potential calculated using equation 2.80 goes to infinity. However we can calculate the potential difference between two points. We have



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$$V = \frac{\lambda}{4\pi\varepsilon_o}\left[ln\left(y' + \sqrt{(y')^2 + r^2}\right)\right]_{-\infty}^{\infty} \quad ...(2.112)$$

The above equation can be written as

$$V = \lim_{t\to\infty} \frac{\lambda}{4\pi\varepsilon_o}\left[ln\left(y' + \sqrt{(y')^2 + r^2}\right)\right]_{-t}^{t} \quad ...(2.113)$$

$$V = \lim_{t\to\infty} \frac{\lambda}{4\pi\varepsilon_o}\left[ln\left(\frac{t + \sqrt{r^2 + t^2}}{-t + \sqrt{r^2 + t^2}}\right)\right] \quad ...2.(114)$$

For t >> r we have

$$\sqrt{r^2 + t^2} \approx t\left[1 + \frac{r^2}{2t^2}\right]$$

$$V = \lim_{t\to\infty} \frac{\lambda}{4\pi\varepsilon_o} ln\left[\frac{t\left(2 + \frac{r^2}{2t^2}\right)}{r^2/2t^2}\right] \quad ...(2.115)$$

Consider a reference point $r_o$. The potential at the reference point $r_o$ is

$$V_o = \lim_{t\to\infty} \frac{\lambda}{4\pi\varepsilon_o} ln\left[\frac{t\left(2 + \frac{r_o^2}{2t^2}\right)}{r_o^2/2t^2}\right] \quad ...(2.116)$$

$$V - V_o = \lim_{t\to\infty} \frac{\lambda}{4\pi\varepsilon_o} ln\left[\frac{t\left(2 + \frac{r^2}{2t^2}\right)}{r^2/2t^2} \cdot \frac{r_o^2/2t^2}{t\left(2 + \frac{r_o^2}{2t^2}\right)}\right] \quad ...(2.117)$$



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$$V - V_o = \lim_{t\to\infty} ln\left[\frac{r_o^2}{r^2}.\frac{\left(2+\frac{r^2}{2t^2}\right)}{\left(2+\frac{r_o^2}{2t^2}\right)}\right] \qquad ...(2.118)$$

Applying the limit

$$V - V_o = \frac{\lambda}{4\pi\varepsilon_o} ln\frac{r_o^2}{r^2} \qquad ...(2.119)$$

$$V - V_o = \frac{\lambda}{2\pi\varepsilon_o} ln\left(\frac{r}{r_o}\right) \qquad ...(2.120)$$

Now the electric field is

$$\mathbf{E} = -\nabla(V - V_o) = -\frac{\partial(V - V_o)}{\partial r} \qquad ...(2.121)$$

$$\mathbf{E} = \frac{\partial}{\partial r}\left[\frac{\lambda}{4\pi\varepsilon_o} ln\left(\frac{r}{r_o}\right)\right] \qquad ...(2.122)$$

$$E = \frac{\lambda}{2\pi\varepsilon_o}\frac{1}{r/r_o}\frac{1}{r_o} \qquad ...(2.123)$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_o r} \qquad ...(2.124)$$

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Why we are using potential difference instead of potential, to calculate **E** and what is the physical significance of the reference point, will be clear to the reader in future sections.



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## 2.14 Electric Field Due to the Finite Line of Charge

Consider a uniformly charged wire of length L, the charge per unit length being λ. Let us calculate the electric field at point P as shown in figure 2.29. The coordinates of point P is ($x$, $y$). Let us divide the entire line of charge into small elements $dx'$ each carrying a charge of $\lambda dx'$. We will calculate the field produced by the small element $\lambda dx'$ and then integrate over the entire length to get the field of the entire line charge. The point of intersection of point P on the line charge is at a distance of $x$ from one end of the line charge and the element $dx'$ is situated at a distance of $x'$ from origin O as shown in figure 2.29. The electric field produced by the element $dx'$ is d**E**. Let us resolve d**E** into $x$ and $y$ components d$\mathbf{E}_x$, d$\mathbf{E}_y$ respectively. From figure 2.29 we can write

$$dE_x = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dx'}{r^2}\cos\theta \qquad ...(2.125)$$

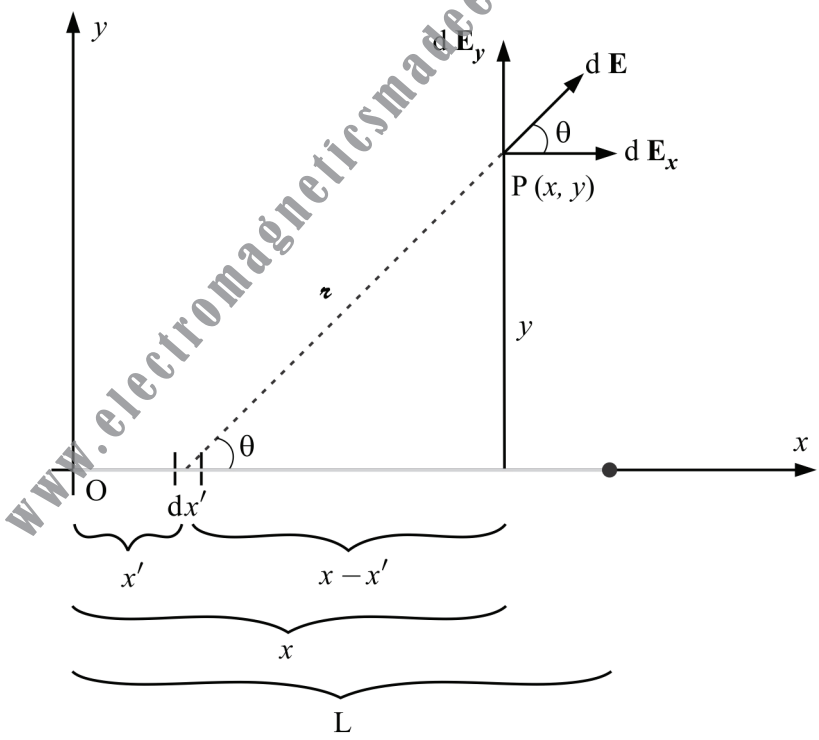


**Fig.2.29**

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But from figure 2.29

$$r^2 = y^2 + (x - x')^2 \qquad ...(2.126)$$

$$\cos\theta = \frac{x - x'}{\sqrt{y^2 + (x - x')^2}} \qquad ...(2.127)$$

Substituting equations 2.126, 2.127 in 2.125

$$dE_x = \frac{1}{4\pi\varepsilon_o} \frac{\lambda dx'}{\left[y^2 + (x - x')^2\right]} \frac{(x - x')}{\sqrt{y^2 + (x - x')^2}} \qquad ...(2.128)$$

$$dE_x = \frac{1}{4\pi\varepsilon_o} \frac{\lambda (x - x') dx'}{\left[y^2 + (x - x')^2\right]^{3/2}} \qquad ...(2.129)$$

The $x$ component of the electric field due to the entire line of charge of length L is

$$E_x = \frac{\lambda}{4\pi\varepsilon_o} \int_0^L \frac{(x - x')dx'}{\left[y^2 + (x - x')^2\right]^{3/2}} \qquad ...(2.130)$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_o} \left[ \frac{1}{\sqrt{y^2 + (x - L)^2}} - \frac{1}{\sqrt{x^2 + y^2}} \right] \qquad ...(2.131)$$

Now let us calculate the $y$-component

$$dE_y = \frac{1}{4\pi\varepsilon_o} \frac{\lambda\, dx'}{r^2} \sin\theta \qquad ...(2.132)$$

From figure 2.29

$$\sin\theta = \frac{y}{\sqrt{y^2 + (x - x')^2}} \qquad ...(2.133)$$

Substituting equations 2.126, 2.133 in equation 2.132



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$$dE_y = \frac{1}{4\pi\varepsilon_o} \frac{\lambda dx'}{\left[y^2 + (x - x')^2\right]} \cdot \frac{y}{\sqrt{y^2 + (x - x')^2}} \quad ...(2.134)$$

$$dE_y = \frac{1}{4\pi\varepsilon_o} \frac{\lambda y dx'}{\left[y^2 + (x - x')^2\right]^{3/2}} \quad ...(2.135)$$

The $y$ component of the electric field due to the entire line of charge of length L is

$$E_y = \frac{\lambda y}{4\pi\varepsilon_o} \int_0^L \frac{dx'}{\left[y^2 + (x - x')^2\right]^{3/2}} \quad ...(2.136)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_o y} \left[ \frac{L - x}{\sqrt{y^2 + (x - L)^2}} + \frac{x}{\sqrt{y^2 + L^2}} \right] \quad ...(2.137)$$

The net electric field at point P due to the entire line charge is then given by

$$\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} \quad ...(2.138)$$

where $E_x$ and $E_y$ are given by equation 2.131 and 2.137 respectively.

**b) Using Gauss's Law**

Step one is to plot the electric field or "to get a feel of the field". For comparison purposes let us sketch the field due to the finite and infinite line of charge.

Initially we show a line charge with two elements $\lambda dx'$ in figure 2.30a. At point P the $\mathbf{E}_y$ components cancel out each other leaving only $\mathbf{E}_x$ components.

Now let us consider the infinite line of charge as shown in figure 2.30b. In this case because the line of charge is infinite, for each line element $\lambda dx'$ there will be always opposite line element $\lambda dx'$ such that $\mathbf{E}_y$ components cancel out and $\mathbf{E}_x$

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components alone adds. Hence either it be point P or $P_1$ or $P_2$ or any other point in figure 2.30b the field is always radial as shown in figure 2.30b because at every point in figure 2.30b the $\mathbf{E}_y$ components will cancel out and $\mathbf{E}_x$ components will add up. Hence radial components are alone present in figure 2.30b. Hence we were able to construct a cylindrical Gaussian surface in figure 2.27b such that the electric field **E**.d**s** can be easily evaluated for the cylindrical Gaussian surface and also **E** is constant over the cylindrical Gaussian surface. Thus using Gauss law we calculated **E** for infinite line of charge in section 2.13b.

Now let us consider finite line of charge. In figure 2.30c we see that at point P the $E_y$ components cancel out and $E_x$ components are alone present. Thus field at point P in figure 2.30c is radial. However at point like $P_1$ and $P_2$ the $E_y$ components will not cancel out. Hence the field will not be radial at points like $P_1$ and $P_2$ as shown in figure 2.30c. Now let us construct a cylindrical Gaussian surface for the finite line of charge as shown in figure 2.31. Clearly at points like P **E**, d**s** are in the same direction. But at points like $P_1$, $P_2$, **E** makes an angle with d**s** as shown in

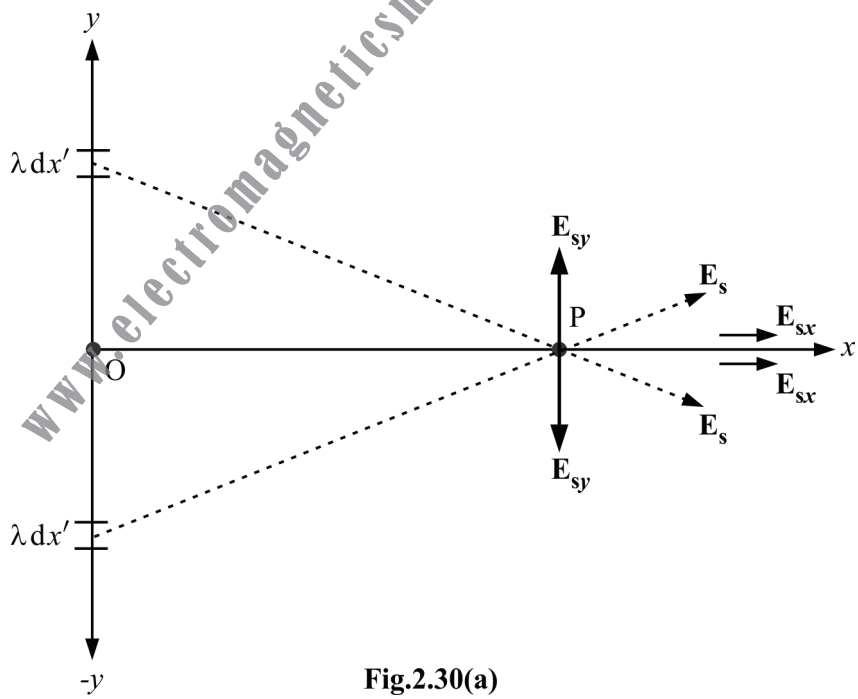


**Fig.2.30(a)**



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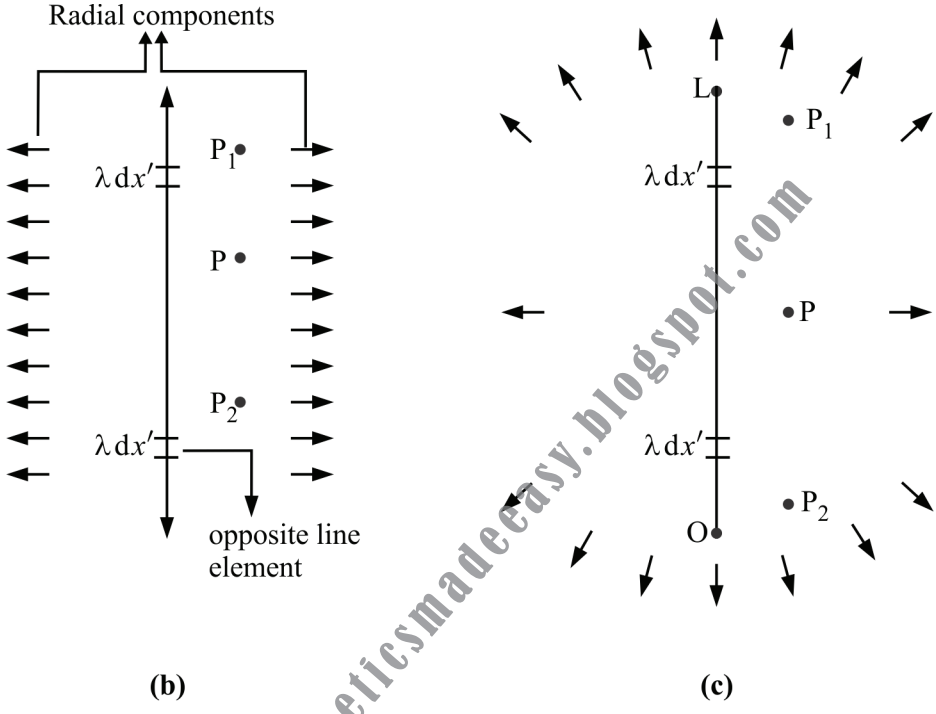


**Fig.2.30**

figure 2.31. Thus at edges of the cylindrical Gaussian surface **E**, d**s** have different directions.

Hence at the edges or at any other point where **E**, d**s** are at some angle to each other evaluation of **E**.d**s** is difficult over the entire Gaussian surface. Even if we some how manage to calculate **E**.d**s**, E cos θ is not constant over the Gaussian surface and hence taking E cos θ out of the integral $\oint \mathbf{E}.d\mathbf{s} = \oint E\cos\theta\, ds$ becomes difficult. It is still true that over the cylindrical Gaussian surface

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

But however evaluation of **E**.d**s** and taking E cos θ out of the integral $\oint \mathbf{E}.d\mathbf{s}$ is difficult. Hence using Gauss's law to calculate electric field in the above case - finite line of charge, becomes tedious. Instead of selecting a cylindrical Gaussian



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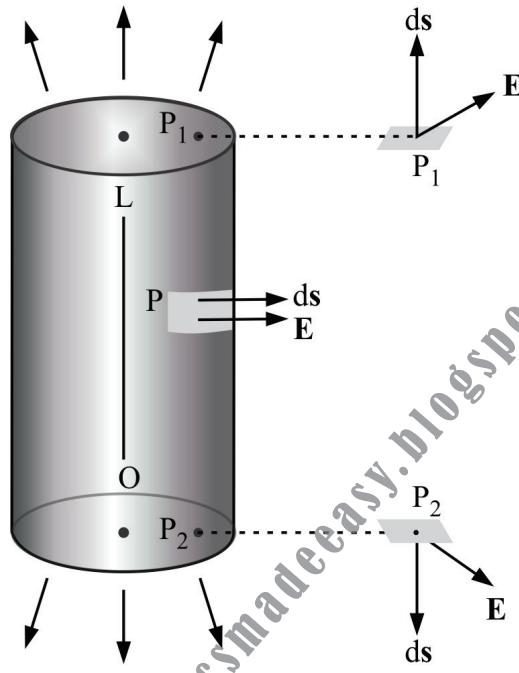


**Fig.2.31**

surface we can select a Gaussian surface of different shape. However until **E**.d**s** can be easily evaluated and E cos θ can be taken out of the integral $\oint \mathbf{E}.\mathbf{ds}$, easy evaluation of **E** using Gauss's law is not possible.

Thus we have seen using Gauss law evaluation of **E**, for a finite line of charge is very difficult.

"The same is true for any charge distribution. For a given charge distribution if we are interested in using Gauss's law to calculate **E**, then we must be able to construct a Gaussian surface such that **E**.d**s** must be evaluated easily and E cos θ must be constant over Gaussian surface so that E cos θ can be pulled out of $\oint \mathbf{E}.\mathbf{ds}$. If we are unable to construct a Gaussian surface which satisfies the above requirement then Gauss's law cannot be easily applied to calculate **E**. It is still true that $\oint \mathbf{E}.\mathbf{ds} = \frac{q_{enc}}{\varepsilon_o}$ over the Gaussian surface, but Gauss law is not useful in calculating **E**.

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**c) Using potential**

The potential at point P in figure 2.29 due to the element $dx'$ which carries a charge of $\lambda dx'$ is

$$dV = \frac{1}{4\pi\varepsilon_o}\frac{\lambda dx'}{r} \qquad ...(2.139)$$

The potential at point P due to the entire line of charge in figure 2.29 is

$$V = \frac{1}{4\pi\varepsilon_o}\int_0^L \frac{\lambda dx'}{r} \qquad ...(2.140)$$

Substituting equation 2.126 in equation 2.140

$$V = \frac{\lambda}{4\pi\varepsilon_o}\int_0^L \frac{dx'}{\sqrt{y^2+(x-x')^2}} \qquad ...(2.141)$$

Integrating

$$V = \frac{\lambda}{4\pi\varepsilon_o}\left[\sinh^{-1}\left(\frac{x-L}{y}\right) - \sinh^{-1}\frac{x}{y}\right] \qquad ...(2.142)$$

Now let us calculate electric field components

$$E_x = \frac{-\partial V}{\partial x} = \frac{\lambda}{4\pi\varepsilon_0}\frac{\partial}{\partial x}\left[\sinh^{-1}\left(\frac{x-L}{y}\right) - \sinh^{-1}\left(\frac{x}{y}\right)\right] \qquad ...(2.143)$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_o}\left[\frac{1}{\sqrt{\left(\frac{x-L}{y}\right)^2+1}}\frac{1}{y} - \frac{1}{\sqrt{\left(\frac{x}{y}\right)^2+1}}\frac{1}{y}\right] \qquad ...(2.144)$$

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$$E_x = \frac{\lambda}{4\pi\varepsilon_o}\left[\frac{1}{\sqrt{(x-L)^2+y^2}} - \frac{1}{\sqrt{x^2+y^2}}\right] \quad ...(2.145)$$

Now the $E_y$ component is

$$E_y = \frac{-\partial V}{\partial y} = \frac{\lambda}{4\pi\varepsilon_o}\frac{\partial}{\partial y}\left[\sinh^{-1}\left(\frac{x-L}{y}\right) - \sinh^{-1}\left(\frac{x}{y}\right)\right] \quad ...(2.146)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_o}\left[\frac{-1}{\sqrt{\frac{(x-L)^2}{y^2}+1}}\frac{x-L}{y^2} - \frac{1}{\sqrt{\frac{x^2}{y^2}+1}}\left(\frac{-x}{y}\right)\right] \quad ...(2.147)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_o y}\left[\frac{L-x}{\sqrt{(x-L)^2+y^2}} + \frac{x}{\sqrt{x^2+y^2}}\right] \quad ...(2.148)$$

Now **E** is given by

$$\mathbf{E} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} \quad ...(2.149)$$

Where $E_x$, $E_y$ the components are given by equations 2.145, 2.148.

## 2.15 Electric Field Along the Axis of a Uniformly Charged Circular Disc

Consider a circular disc of radius R on which charge is uniformly distributed with surface charge density $\sigma$. Let us calculate the electric field at point P at a distance *y* from the center of the disc as shown in figure 2.32.

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**a) Using Coulomb's Law**

We have calculated the electric field for the above problem in example 2.7 using Coulomb's law.

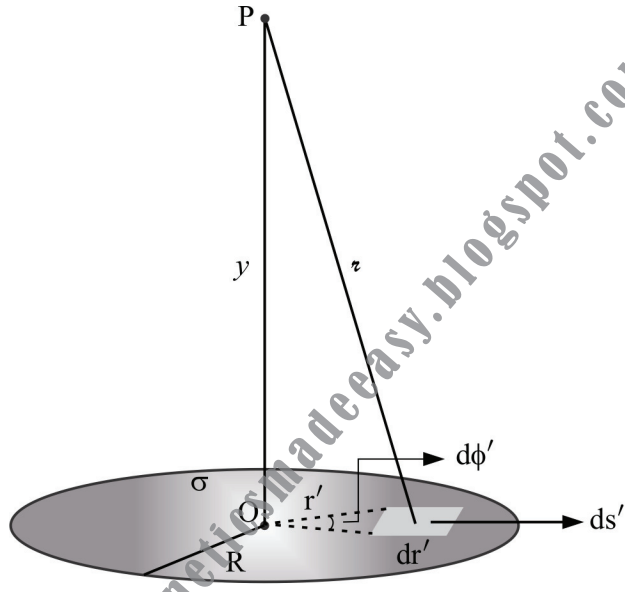


**Fig.2.32**

**b) Using Gauss's Law**

In figure 2.33 field distribution for a uniformly charged disc at few points is shown. Clearly construction of a Gaussian surface (over which **E**.d**s** can be easily evaluated) is not that easy. Hence for similar reasons mentioned in section 2.14b Gauss law is not useful here in calculating **E**.

**c) Using Potential**

Let us calculate the electric field at point P in figure 2.32 using potential. Consider a surface element $ds'$ situated at a distance of $r$ from point P. The potential due to the surface element at point P is given by

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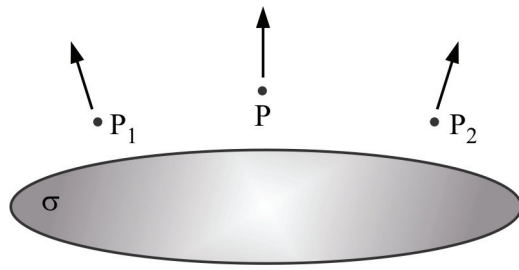


**Fig.2.33**

$$V = \frac{1}{4\pi\varepsilon_o} \int_{S'} \frac{\sigma ds'}{r} \qquad ...(2.150)$$

From figure 2.32

$$ds' = r'\, dr'\, d\phi' \qquad ...(2.151)$$

$$r = \sqrt{y^2 + (r')^2} \qquad ...(2.152)$$

Substituting equations 2.151, 2.152 in equation 2.150

$$V = \frac{1}{4\pi\varepsilon_o} \int_0^{2\pi} \int_0^{R} \frac{\sigma\, r'\, dr'\, d\phi'}{\sqrt{y^2 + (r')^2}} \qquad ...(2.153)$$

$$V = \frac{2\pi\sigma}{4\pi\varepsilon_o} \int_0^{R} \frac{r'\, dr'}{\sqrt{y^2 + (r')^2}} \qquad ...(2.154)$$

$$V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{y^2 + R^2} - y\right] \qquad ...(2.155)$$

$$\mathbf{E} = -\nabla V = -\left[\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y}\right]\left[\frac{\sigma}{2\varepsilon_o}\left(\sqrt{y^2 + R^2} - y\right)\right] \qquad ...(2.156)$$

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As $\frac{\partial V}{\partial x} = 0$

$$\mathbf{E} = -\frac{\partial}{\partial y}\left[\frac{\sigma}{2\varepsilon_o}\left(\sqrt{y^2 + R^2} - y\right)\right]\hat{\mathbf{j}} \qquad ...(2.157)$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{y}{\sqrt{y^2 + R^2}}\right]\hat{\mathbf{j}} \qquad ...(2.158)$$

**Note:**

Integration here is carried from 0 to $2\pi$ for $\phi'$, that is over the entire disc because we consider only one surface element $ds'$. In example 2.7 we carry out integration from 0 to $\pi$ for $\phi'$, that is over only half of the disc because we consider two surface elements.

## 2.16 Electric Field Due to an Inifinite Plane Sheet of Charge

Consider a infinite plane sheet of charge carrying a uniform surface charge density $\sigma$ as shown in figure 2.34. We are interested in calculating the electric field at point P. We will use cylindrical coordinates (r, $\phi$, $z$). Consider two strips $ds'$ situated opposite to each other. The electric field $\mathbf{E_s}$ due to each strip is shown in the figure. Resolving the $\mathbf{E_s}$ into components $\mathbf{E}_{sr}$, $\mathbf{E}_{sz}$ we see that $\mathbf{E}_{sr}$ components cancel out while $\mathbf{E}_{sz}$ components add up.

Hence the net field due to the two strips at point P is

$$d\mathbf{E} = 2\, E_{sz}\, \hat{\mathbf{z}} \qquad ...(2.159)$$

From figure 2.34

$$\sin\theta = \frac{E_{sz}}{E_s}$$

$$\Rightarrow E_{sz} = E_s \sin\theta \qquad ...(2.160)$$

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The field due to single strip $ds'$ at point P is

$$E_s = \frac{1}{4\pi\varepsilon_o}\frac{\sigma ds'}{r^2} \qquad ...(2.161)$$

From figure 2.34

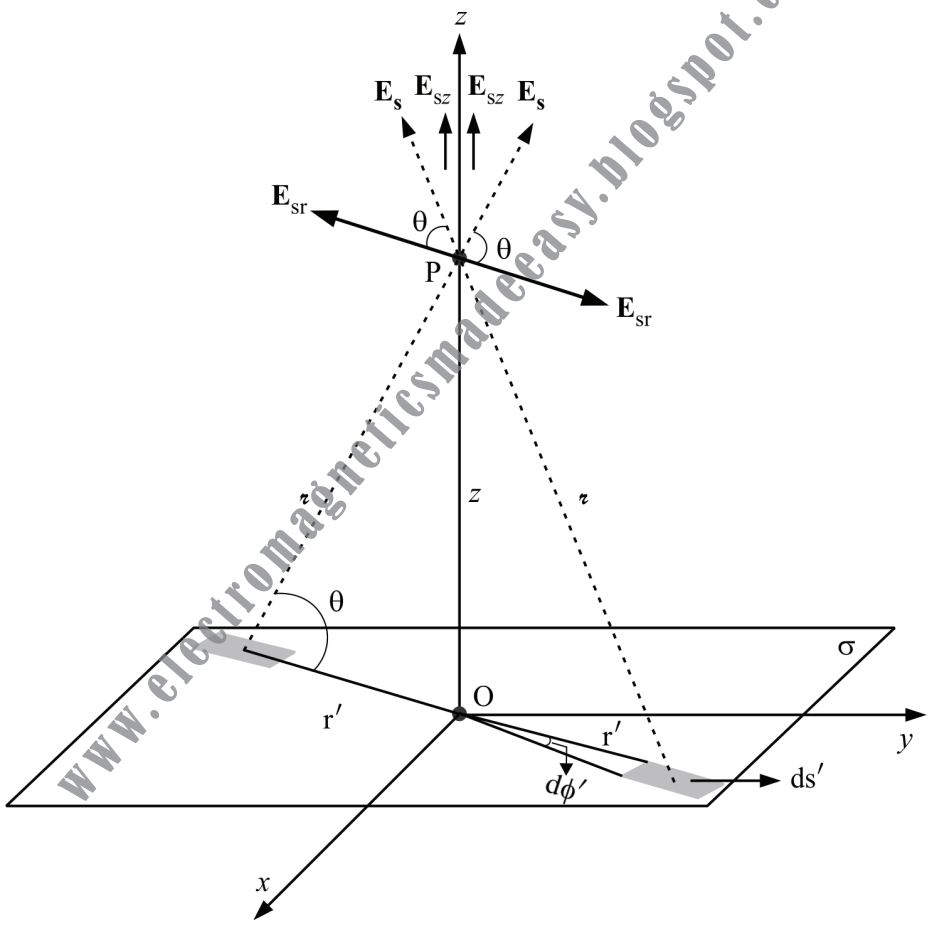


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**Fig.2.34**



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$$\sin\theta = \frac{z}{r}$$

But $r = \sqrt{z^2 + (r')^2}$ ...(2.162)

Hence $\sin\theta = \dfrac{z}{\sqrt{z^2 + (r')^2}}$ ...(2.163)

Substituting equations 2.161, 2.162, 2.163 in equation 2.159

$$d\mathbf{E} = 2E_s \sin\theta\, \hat{\mathbf{z}}$$

$$d\mathbf{E} = 2\frac{1}{4\pi\varepsilon_o}\frac{\sigma ds'}{r^2}\sin\theta\, \hat{\mathbf{z}} \quad ...(2.164)$$

$$d\mathbf{E} = 2\frac{1}{4\pi\varepsilon_o}\frac{\sigma ds'}{\left[z^2 + (r')^2\right]}\frac{z}{\sqrt{z^2 + (r')^2}}\hat{\mathbf{z}} \quad ...(2.165)$$

$$d\mathbf{E} = \frac{\sigma z}{2\pi\varepsilon_o}\frac{ds'}{\left[z^2 + (r')^2\right]^{3/2}}\hat{\mathbf{z}} \quad ...(2.166)$$

The field due to the entire plane sheet of charge is then

$$\mathbf{E} = \frac{\sigma z}{2\pi\varepsilon_o}\int_{S'}\frac{ds'}{\left[z^2 + (r')^2\right]^{3/2}}\hat{\mathbf{z}} \quad ...(2.167)$$

$$\mathbf{E} = \frac{\sigma z}{2\pi\varepsilon_o}\int_0^{\pi}\int_0^{\infty}\frac{r'\, dr'\, d\phi'}{\left[(r')^2 + z^2\right]^{3/2}}\hat{\mathbf{z}} \quad ...(2.168)$$

$$\mathbf{E} = \frac{\sigma z}{2\varepsilon_o}\int_0^{\infty}\frac{r' dr'}{\left[(r')^2 + z^2\right]^{3/2}}\hat{\mathbf{z}} \quad ...(2.169)$$

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$$\mathbf{E} = \frac{\sigma z}{2\varepsilon_o}\left[\frac{-1}{\left((r')^2 + z^2\right)^{1/2}}\right]_0^{\infty} \hat{\mathbf{z}} \qquad ...(2.170)$$

$$\mathbf{E} = \frac{\sigma z}{2\,\varepsilon_o}\,\frac{1}{z}\,\hat{\mathbf{z}} \qquad ...(2.171)$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_o}\,\hat{\mathbf{z}} \qquad ...(2.172)$$

**b) Using Gauss's Law**

Now let us calculate the electric field of a infinite plane sheet of charge using Gauss's law. Step one is to get feel of the field. Clearly as shown in figure 2.34, 2.35a the field due to a infinite plane sheet of charge points in *z* direction.

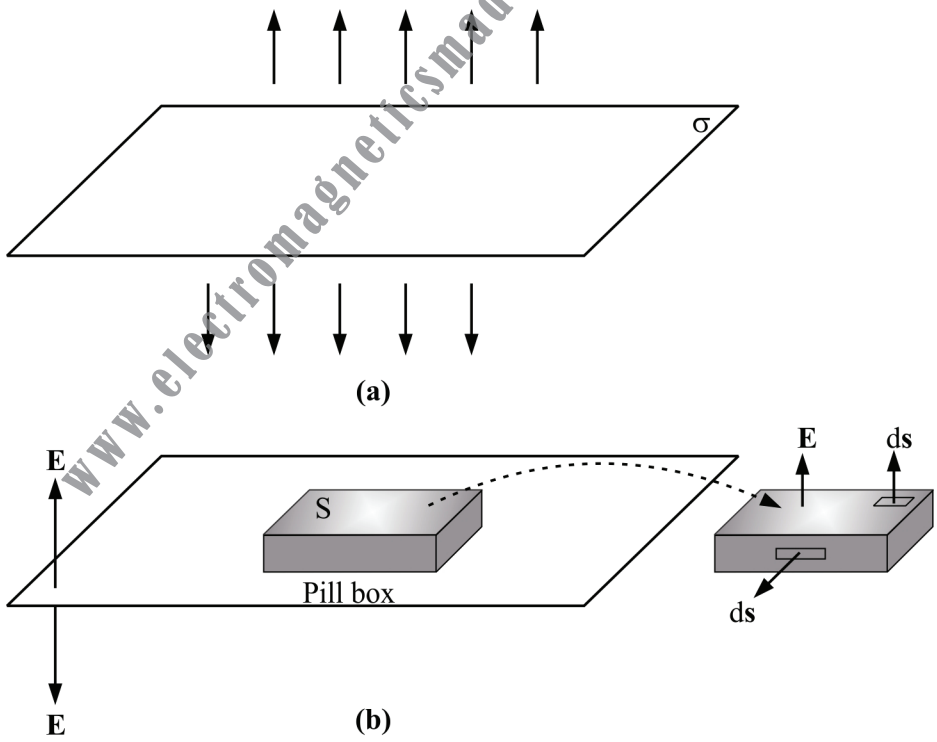


**Fig.2.35**



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Hence as shown in figure 2.35b we construct a Gaussian surface in the shape of pill box with surface area S. Half of the pill box lies above the plane sheet of charge and half of the pill box lies below the plane sheet of charge. Bottom half of the pill box is not shown in the figure.

As per Gauss's law

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

The above integral can be written for the pill box as

$$\left|\int \mathbf{E}.d\mathbf{s}\right|_{top} + \left|\int \mathbf{E}.d\mathbf{s}\right|_{bottom} + \left|\int \mathbf{E}.d\mathbf{s}\right|_{sides} = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.173)$$

For the sides of two pill box as shown in figure 2.35b the angle between **E**, d**s** is 90°. Hence **E**.d**s** = 0 for sides of the pill box. Equation 2.173 is then

$$\left|\int \mathbf{E}.d\mathbf{s}\right|_{top} + \left|\int \mathbf{E}.d\mathbf{s}\right|_{bottom} = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.174)$$

For the top and bottom of pill box the angle between **E**, d**s** is zero. Hence

$$\left|\int_S |\mathbf{E}|\,|d\mathbf{s}|\cos 0\right|_{top} + \left|\int_S |\mathbf{E}|\,|d\mathbf{s}|\cos 0\right|_{bottom} = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.175)$$

$$\Rightarrow \int_S |\mathbf{E}|\,|d\mathbf{s}| + \int_S |\mathbf{E}|\,|d\mathbf{s}| = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.176)$$

Because |**E**| is constant over the top and bottom faces of the pill box

$$|\mathbf{E}| \int_S |d\mathbf{s}| + |\mathbf{E}| \int_S |d\mathbf{s}| = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.177)$$

$$E\,S + E\,S = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.178)$$

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From figure 2.35 $q_{enc} = \sigma S$. Hence,

$$2\,E\,S = \frac{\sigma S}{\varepsilon_o} \qquad ...(2.179)$$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_o} \qquad ...(2.180)$$

$$\Rightarrow \mathbf{E} = \frac{\sigma}{2\varepsilon_o}\,\hat{\mathbf{z}} \qquad ...(2.181)$$

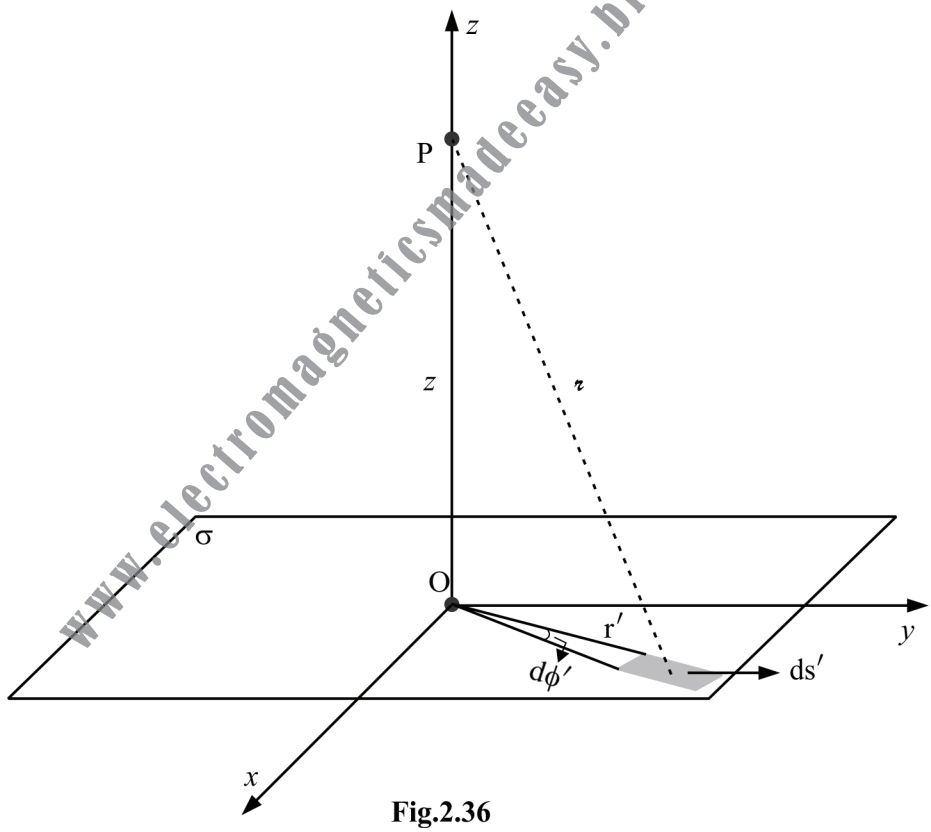


**Fig.2.36**

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**c) Using Potential**

The potential at point P due to entire plane in figure 2.36 is

$$V = \frac{1}{4\pi\varepsilon_o}\int_{S'}\frac{\sigma\, ds'}{r} \qquad ...(2.182)$$

$$V = \frac{\sigma}{4\pi\varepsilon_o}\int_0^{2\pi}\int_0^{\infty}\frac{r'\, dr'\, d\phi'}{\sqrt{(r')^2 + z^2}} \qquad ...(2.183)$$

by substituting equation 2.162.

$$V = \frac{\sigma}{2\varepsilon_o}\int_0^{\infty}\frac{r'\, dr'}{\sqrt{(r')^2 + z^2}} \qquad ...(2.184)$$

The above equation can be written as

$$V = \lim_{t\to\infty}\frac{\sigma}{2\varepsilon_o}\int_0^{t}\frac{r'\, dr'}{\sqrt{(r')^2 + z^2}} \qquad ...(2.185)$$

$$V = \lim_{t\to\infty}\frac{\sigma}{2\varepsilon_o}\left[\sqrt{(r')^2 + z^2}\right]_0^t \qquad ...(2.186)$$

$$V = \lim_{t\to\infty}\frac{\sigma}{2\varepsilon_o}\left[\sqrt{t^2 + z^2} - z\right] \qquad ...(2.187)$$

As $t >> z$, $\qquad \sqrt{t^2 + z^2} \approx t\left[1 + \frac{z^2}{2t^2}\right]$

Hence

$$V = \lim_{t\to\infty}\frac{\sigma}{2\varepsilon_o}\left[t\left[1 + \frac{z^2}{2t^2}\right] - z\right] \qquad ...(2.188)$$

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$$V = \lim_{t\to\infty} \frac{\sigma}{2\varepsilon_o}\left[t + \frac{z^2}{2t} - z\right] \quad ...(2.189)$$

Consider a reference point $z_0$. The potential at reference point $z_0$ is

$$V_o = \lim_{t\to\infty} \frac{\sigma}{2\varepsilon_o}\left[t + \frac{z_o^2}{2t} - z_o\right] \quad ...(2.190)$$

The potential difference between points $z$ and $z_0$ is

$$V - V_o = \frac{\sigma}{2\varepsilon_o} \lim_{t\to\infty}\left[\frac{z^2}{2t} - \frac{z_o^2}{2t} + z_o - z\right] \quad ...(2.191)$$

Applying the limit

$$V - V_o = \frac{\sigma}{2\varepsilon_o}\left[z_o - z\right] \quad ...(2.192)$$

Now the electric field is

$$\mathbf{E} = -\nabla(V - V_o) \quad ...(2.193)$$

In cylindrical coordinates

$$\mathbf{E} = -\left[\frac{\partial(V - V_o)}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial(V - V_o)}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial(V - V_o)}{\partial z}\hat{\mathbf{z}}\right] \quad ...(2.194)$$

$$E = -\frac{\partial(V - V_o)}{\partial z}\hat{\mathbf{z}} \quad ...(2.195)$$

as $V - V_o$ in equation 2.192 doesn't contain terms with r, $\phi$.

Substituting equation 2.192 in 2.195

$$E = \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{z}} \quad ...(2.196)$$

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Why we use potential difference instead of potential to calculate electric field and what is the physical significance of reference point $z_o$ will be clear to the reader in future sections.

## 2.17 Electric Field of a Uniformly Charged Spherical Shell

Consider a spherical shell of radius R as shown in figure 2.37. Charge q is distributed uniformly on the shell with surface charge density σ.

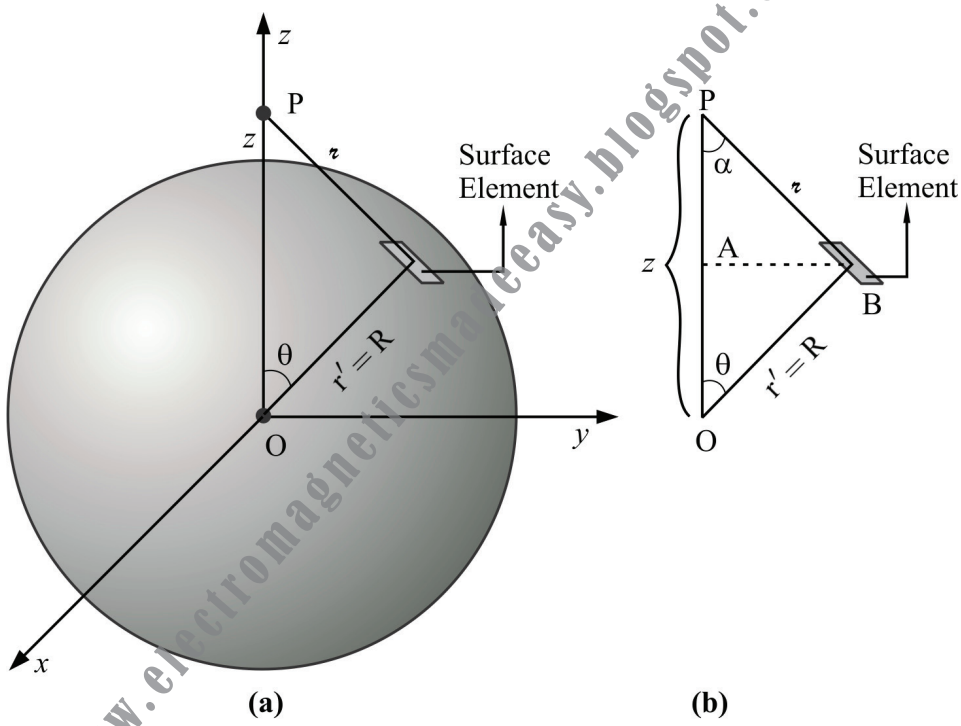


**Fig.2.37**

**a) Using Coulomb's Law**

***i) At a point interior to the spherical shell***

Consider point P in the interior of the spherical shell as shown in figure 2.38. Consider two cones with pont P as their apex extending upto the surface of the sphere where they cut the sphere. Cone 1 and cone 2 substends a solid angle of $d\Omega_1$ and $d\Omega_2$ respectively. Both the solid angles are equal



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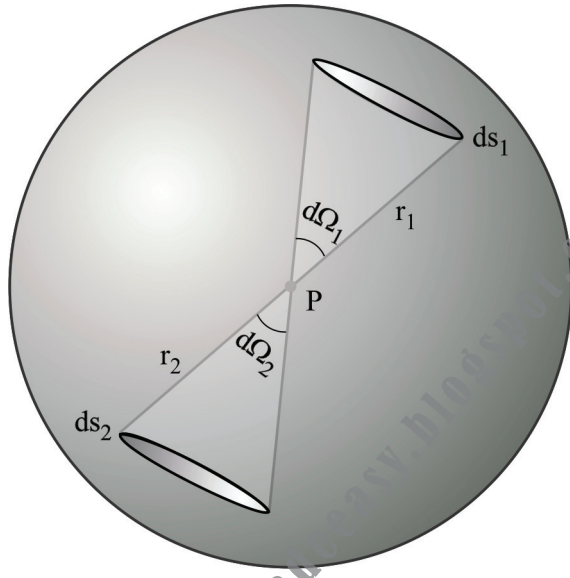


**Fig.2.38**

$$d\Omega_1 = d\Omega_2 \qquad ...(2.197)$$

$$\Rightarrow \frac{ds_1}{r_1^2} = \frac{ds_2}{r_2^2} \qquad ...(2.198)$$

$$\Rightarrow \frac{ds_2}{ds_1} = \frac{r_2^2}{r_1^2} \qquad ...(2.199)$$

Because the spherical shell is uniformly charged the charge on the each elements is proportional to the area. Hence,

$$\frac{dq_2}{dq_1} = \frac{ds_2}{ds_1} \qquad ...(2.200)$$

The ratio between electric field at point P due to the elements $ds_1$ and $ds_2$ is

$$\frac{E_2}{E_1} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{dq_2}{r_2^2}}{\frac{1}{4\pi\varepsilon_0}\frac{dq_1}{r_1^2}} \qquad ...(2.201)$$



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$$\frac{E_2}{E_1} = \frac{dq_2}{dq_1}\frac{r_1^2}{r_2^2} \qquad ...(2.202)$$

From equations 2.199, 2.200

$$\frac{dq_2}{dq_1} = \frac{r_2^2}{r_1^2} \qquad ...(2.203)$$

Hence

$$\frac{E_2}{E_1} = 1 \qquad ...(2.204)$$

Thus the field due to the elements $ds_1$ and $ds_2$ at point P are equal in magnitude. From figure 2.38 the field due to $ds_1$ and $ds_2$ at point P is oppositely directed. Hence the field at point P is zero. This is true for any point in the interior of the sphere.

***ii) At a point exterior to the spherical shell***

Consider figure 2.37a. Clearly the *x*, *y* components of electric field at point P due to the symmetrically placed surface elements cancel out and only the *z* components add up. Thus we have to integrate over the *z* components, $dE_s \cos\alpha$ of the elements and here $dE_s$ is field produced by the surface element at P given by

$$dE_s = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2} \qquad ...(2.205)$$

We have

$$dq = \sigma\, ds' = \sigma\, R^2 \sin\theta'\, d\theta'\, d\phi' \qquad ...(2.206)$$

in spherical polar coordinates.

From figure 2.37b

$$r^2 = R^2 + z^2 - 2Rz\cos\theta' \qquad ...(2.207)$$

($\theta$ in figure 2.37b is denoted as $\theta'$ in equation 2.207 to note that it is a source point).



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Also,

$$\cos\alpha = \frac{z - R\cos\theta'}{r} \qquad ...(2.208)$$

Now the electric field at point P due to the entire shell is

$$E_z = \int dE_s \cos\alpha \qquad ...(2.209)$$

Substituting equation 2.205 in equation 2.209

$$E_z = \int \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\cos\alpha \qquad ...(2.210)$$

Substituting equations 2.206, 2.207 and 2.208 in equation 2.210

$$E_z = \frac{\sigma R^2}{4\pi\varepsilon_o}\int_0^{2\pi} d\phi' \int_0^{\pi} \frac{(z - R\cos\theta')\sin\theta'\, d\theta'}{(R^2 + z^2 - 2Rz\cos\theta')^{3/2}} \qquad ...(2.211)$$

$$E_z = \frac{2\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\int_0^{\pi} \frac{(z - R\cos\theta')\sin\theta'\, d\theta'}{(R^2 + z^2 - 2Rz\cos\theta')^{3/2}} \qquad ...(2.212)$$

Let $t = \cos\theta' \quad dt = -\sin\theta'\, d\theta'$

$\theta' = 0 \Rightarrow t = 1$

$\theta' = \pi \Rightarrow t = -1$

Hence

$$E_z = \frac{2\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\left[\int_{-1}^{1} \frac{(z - Rt)\,dt}{(R^2 + z^2 - 2R\,zt)^{3/2}}\right] \qquad ...(2.213)$$

$$E_z = \frac{2\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\left[\frac{1}{z^2}\frac{(zt - R)}{\sqrt{R^2 + z^2 - 2R\,zt}}\right]_{-1}^{1} \qquad ...(2.214)$$



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$$E_z = \frac{2\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\left[\frac{1}{z^2}\left(\frac{z-R}{\sqrt{(z-R)^2}}+\frac{z+R}{\sqrt{(z+R)^2}}\right)\right] \qquad ...(2.215)$$

$$E_z = \frac{2\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\left[\frac{2}{z^2}\right] = \frac{4\pi\,\sigma\,R^2}{4\pi\varepsilon_o}\;\frac{1}{z^2} \qquad ...(2.216)$$

But $4\pi\,R^2\,\sigma = q$ the total charge enclosed by the spherical surface. Hence

$$\mathbf{E}_z = \frac{1}{4\pi\varepsilon_o}\,\frac{q}{z^2}\,\hat{\mathbf{k}} \qquad ...(2.217)$$

Now the total electric field at point P due to the entire spherical shell is

$$\mathbf{E} = E_x\,\hat{\mathbf{i}} + E_y\,\hat{\mathbf{j}} + E_z\,\hat{\mathbf{k}} = \frac{1}{4\pi\varepsilon_o}\,\frac{q}{z^2}\,\hat{\mathbf{k}}$$

as $E_x$ and $E_y$ are zero.

**b) Using Gauss's Law**

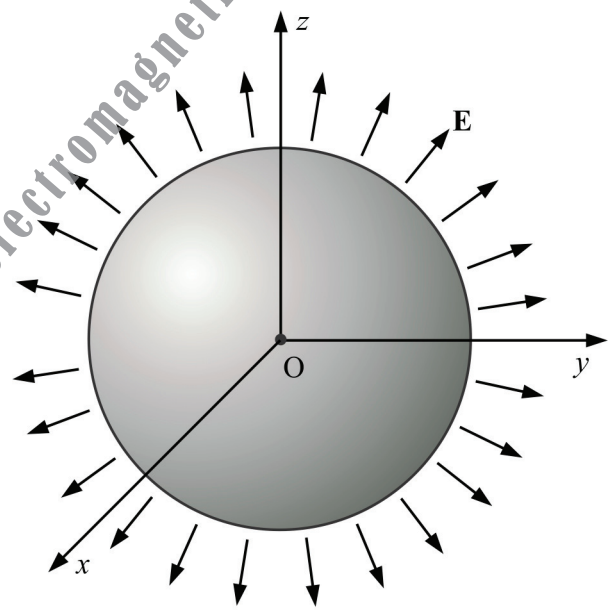


**Fig.2.39**



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## Electrostatics

First step is to plot the electric field of the uniformly charged spherical shell as shown in figure 2.39. Clearly if we want to evaluate **E**.d**s** easily then we have to construct the Gaussian surface to be a sphere.

***i) At a point interior to the spherical shell***

We construct a Gaussian surface as shown in figure 2.40 for a interior point P. Gauss law states

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$$

From figure 2.40 the angle between **E**, d**s** is zero and the charge $q_{enc}$ enclosed by the Gaussian surface is zero. Hence from Gauss's law

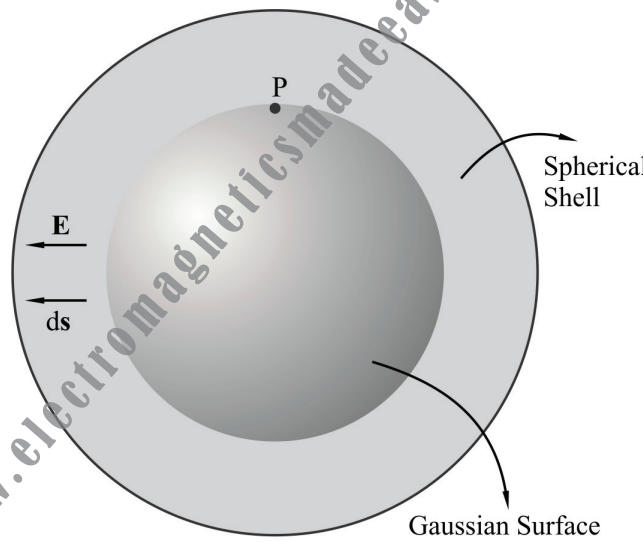


**Fig.2.40**

$$\int |\mathbf{E}|\,|d\mathbf{s}| \cos 0 = 0 \qquad \ldots(2.218)$$

$$\Rightarrow |\mathbf{E}| = 0$$



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Hence electric field at the interior point of the charged spherical shell is zero.

***ii) At a point exterior to the spherical shell***

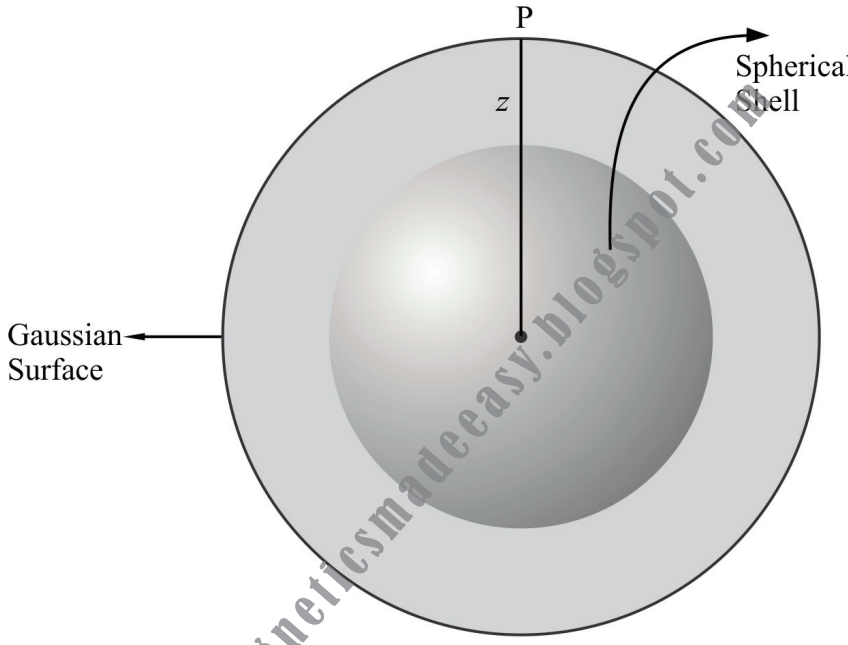


**Fig.2.41**

We construct a Gaussian surface for an exterior point P as shown in figure 2.41. The angle between **E** and d**s** is zero and the total charge enclosed by the spherical shell is q. Applying Gauss' s law

$$\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o} \qquad ...(2.219)$$

$$\oint |\mathbf{E}|\,|d\mathbf{s}| \cos 0 = \frac{q}{\varepsilon_o} \qquad ...(2.220)$$

$$\Rightarrow |\mathbf{E}| \oint |d\mathbf{s}| = \frac{q}{\varepsilon_o} \qquad ...(2.221)$$

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$$\Rightarrow |\mathbf{E}|\, 4\pi z^2 = \frac{q}{\varepsilon_o} \qquad ...(2.222)$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_o z^2} \qquad ...(2.223)$$

**c) Using Potential**

The potential at point P in figure 2.37a is given by

$$V = \frac{1}{4\pi\varepsilon_o}\int \frac{\sigma\, ds'}{r} \qquad ...(2.224)$$

Substituting equations 2.207 and $ds'$ we get

$$V = \frac{1}{4\pi\varepsilon_o}\sigma\int \frac{R^2 \sin\theta'\, d\theta'\, d\phi'}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}} \qquad ...(2.225)$$

$$V = \frac{1}{4\pi\varepsilon_o}\int_0^{2\pi} d\phi' \int_0^{\pi} \frac{R^2 \sin\theta'\, d\theta'}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}} \qquad ...(2.226)$$

$$V = \frac{2\pi R^2\sigma}{4\pi\varepsilon_o}\int_0^{\pi} \frac{\sin\theta'\, d\theta'}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}} \qquad ...(2.227)$$

$$V = \frac{2\pi R^2\sigma}{4\pi\varepsilon_o}\left[\frac{1}{Rz}\sqrt{R^2 + z^2 - 2Rz\cos\theta'}\right]_0^{\pi} \qquad ...(2.228)$$

$$V = \frac{2\pi R^2\sigma}{4\pi\varepsilon_o Rz}\left[\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right] \qquad ...(2.229)$$

***i) Inside the spherical shell***

Inside the spherical shell $z < R$.

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Hence equation 2.229 is

$$V = \frac{2\pi R^2 z}{4\pi\varepsilon_o Rz}\left[(R+z)-(R-z)\right] \qquad ...(2.230)$$

$$V = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_o R} \qquad ...(2.231)$$

$$V = \frac{q}{4\pi\varepsilon_o R} \qquad ...(2.232)$$

Now $\mathbf{E} = -\nabla V$

$$= -\left[\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right]\frac{q}{4\pi\varepsilon_o R}$$

$$\mathbf{E} = 0$$

***ii) Outside the Spherical Shell***

Outside the spherical shell $z > R$

Hence equation 2.229 is

$$V = \frac{2\pi R^2 \sigma}{4\pi\varepsilon_o Rz}\left[\sqrt{(R+z)^2} - \sqrt{(z-R)^2}\right] \qquad ...(2.233)$$

$$V = \frac{2\pi R^2 \sigma}{4\pi\varepsilon_o Rz}\left[(R+z)-(z-R)\right] \qquad ...(2.234)$$

$$V = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_o z} = \frac{q}{4\pi\varepsilon_o z} \qquad ...(2.235)$$

Now $\mathbf{E} = -\nabla V$

$$= -\left[\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right]\frac{q}{4\pi\varepsilon_o z} \qquad ...(2.236)$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_o z^2}\hat{\mathbf{k}} \qquad ...(2.237)$$

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Of the three methods discussed here Gauss's law is the easiest as the reader may observe.

The next two methods are Coulomb's law and potential formulation. Luckily in this problem $E_x$, $E_y$ components cancelled out and $E_z$ component alone exist and the reader can see how much steps are involved in calculating $E_z$ component using Coulomb's law. Suppose for a given charge distribution in addition to $E_z$ component if $E_x$ and $E_y$ components do not cancel out then $E_x$ and $E_y$ must be calculated separately. That is, as we calculated $E_z$ through equations 2.205 to 2.217 we have to repeat similar procedure for $E_x$, $E_y$ each separately. The reader can see how much tedious procedure it would be. The problem is, Coulomb's law calculates electric field directly in terms of vector. There are three different directions $x$, $y$, $z$ and hence three components $E_x$, $E_y$, $E_z$. However the potential V is a scalar quantity and we have already seen that it doesn't have components. So we perform only single integration as we have done in equations 2.224 to 2.229 and finally calculate E as

$$\mathbf{E} = -\nabla \mathrm{V} = -\left[\hat{\mathbf{i}}\frac{\partial \mathrm{V}}{\partial x} + \hat{\mathbf{j}}\frac{\partial \mathrm{V}}{\partial y} + \hat{\mathbf{k}}\frac{\partial \mathrm{V}}{\partial z}\right]$$

$$\mathbf{E} = \mathrm{E}_x\,\hat{\mathbf{i}} + \mathrm{E}_y\hat{\mathbf{j}} + \mathrm{E}_z\,\hat{\mathbf{k}}$$

which is easy as compared to Coulomb's Law.

## 2.18 Comparison of Coulomb's Law, Gauss's Law and Potential Formulation

Let us now compare the methods we used to calculate the electric field for a given charge distribution - Coulomb's law, Gauss law, potential formulation.

Out of the three methods we see that Gauss's law is the easiest method. From selections 2.13, 2.16 and 2.17 we observe Gauss's law is able to calculate the electric field intensities for the respective charge distributions very easily as compared to Coulomb's law and potential formulation.

Successful application of Gauss's law requires construction of Gaussian surface for a given charge distribution over which **E**.d**s** can be easily evaluated and



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over the Gausian surface |**E**| cos θ must be constant so that |**E**| cos θ can be pulled out of the integral

$$\oint \mathbf{E}.d\mathbf{s} = \oint |\mathbf{E}| \cos\theta \, ds = |\mathbf{E}| \cos\theta \oint ds$$

Evaluation of the area $\oint ds$ is simple mathematics. E can thus be calculated using Gauss law

$$\oint \mathbf{E}.d\mathbf{s} = |\mathbf{E}| \cos\theta \oint ds = \frac{q_{enc}}{\varepsilon_o}$$

$$|\mathbf{E}| = \frac{1}{\cos\theta \oint ds} \frac{q_{enc}}{\varepsilon_o}$$

The above conditions are easily satisfied when symmetry exists in the given charge distributions. In section 2.13 cylindrical symmetry exists. In section 2.16 planar symmetry exists. In section 2.17 spherical symmetry exists. Existence of symmetries in the respective charge distributions facilitated easy calculation of $\oint \mathbf{E}.d\mathbf{s}$.

If symmetry doesn's exists in the given problem then easy calculation of **E**.d**s** is not possible using Gauss's law. In such situations Gauss's law is not useful in calculation of **E** although it is still true that $\oint \mathbf{E}.d\mathbf{s} = \frac{q_{enc}}{\varepsilon_o}$.

This point has been made clear in sections 2.14, 2.15.

When Gauss' law is not useful we are left out with two options to calculate the electric field - Coulomb's law and potential formulation.

As we have previouly mentioned Coulomb's law calculates **E** directly in terms of vector. This makes the calculation of **E** using Coulomb's law difficult. Section 2.17 illustrates this point. If you are interested in calculated **E** using Coulomb's law then you have to work with three components - $E_x$, $E_y$, $E_z$. However working with potential is easy. Potential is a scalar quantity and doesn't have to be split into components.

Working with vectors is comparatively difficult than working with scalars. This fact has been made clear in section 1.1 and example 1.10. This point has also been made clear in section 2.17.



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We will see one more example below to show working with vectors is diffcult as compared to working with scalars.

We redraw figure 2.11a, 2.32 in figure 2.42a, b.

In figure 2.42a the electric fields due to two surface elements $ds_1'$, $ds_2'$ is shown. The electric field produced by the surface elements at A, B acts along AC and BD respectively. These fields are denoted as $\mathbf{E}_1$ and $\mathbf{E}_2$ respectively. Resolving $\mathbf{E}_1$ and $\mathbf{E}_2$ as shown in figure 2.43 into $x$ and $y$ components we see that $x$ components cancel out while $y$ components add up.

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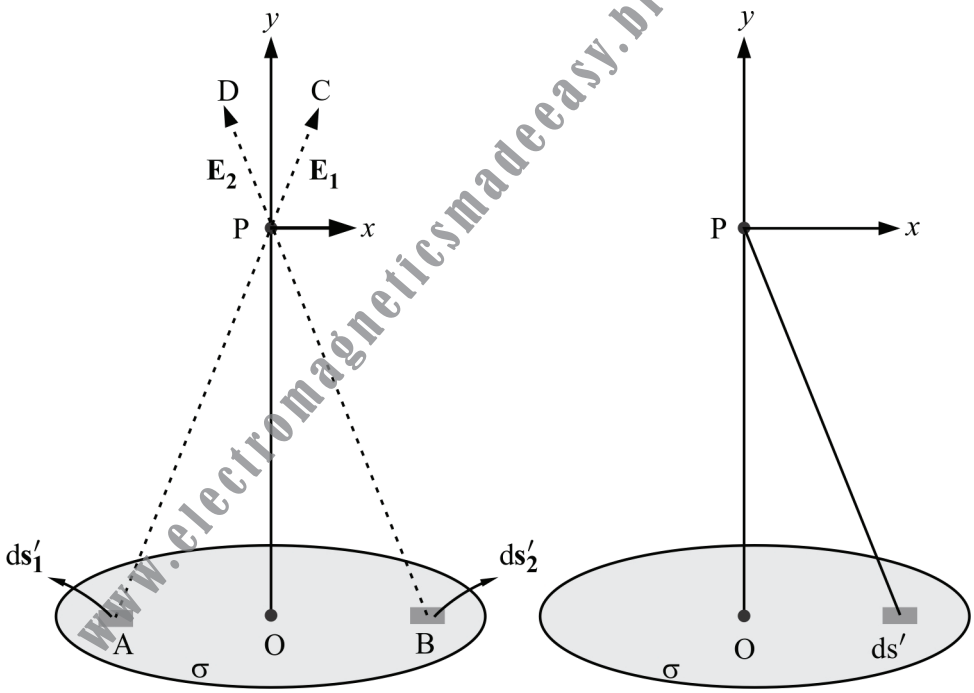


**Fig.2.42**



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Then we need to integrate the $E_y$ components to calculate the net electric at point P due to entire disc.

Now consider figure 2.44a.

We are interested in calculating the electric field at point $P_1$ due to the charge density $\sigma$ over the entire disc, we have resolved the electric fields, $\mathbf{E_1}$, $\mathbf{E_2}$ due

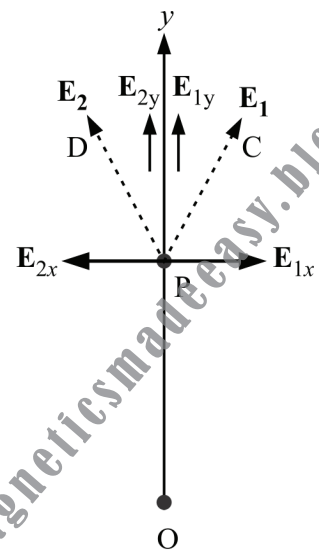


**Fig.2.43**

to the elements $d\mathbf{s_1}$, $d\mathbf{s_2}$ into *x*, *y* components in figure 2.44 b. Clearly both *x*, *y* components are adding up. $E_{1x}$ and $E_{2x}$ are in same direction and $E_{1y}$ and $E_{2y}$ are in same direction. Summing up we get *x* and *y* components due to the two elements. Then we integrate over the entire disc to get $E_x$ and $E_y$ and hence the electric field.

If the problem happens in three dimensions then we need to keep track of three components $E_x$, $E_y$, $E_z$ as we have mentioned at the end of section 2.17.

The trouble with above method is electric field being a vector has a direction. For example in figure 2.44a $\mathbf{E_1}$ is in the direction from $P_1$ to $C_1$ while $\mathbf{E_2}$ is in the



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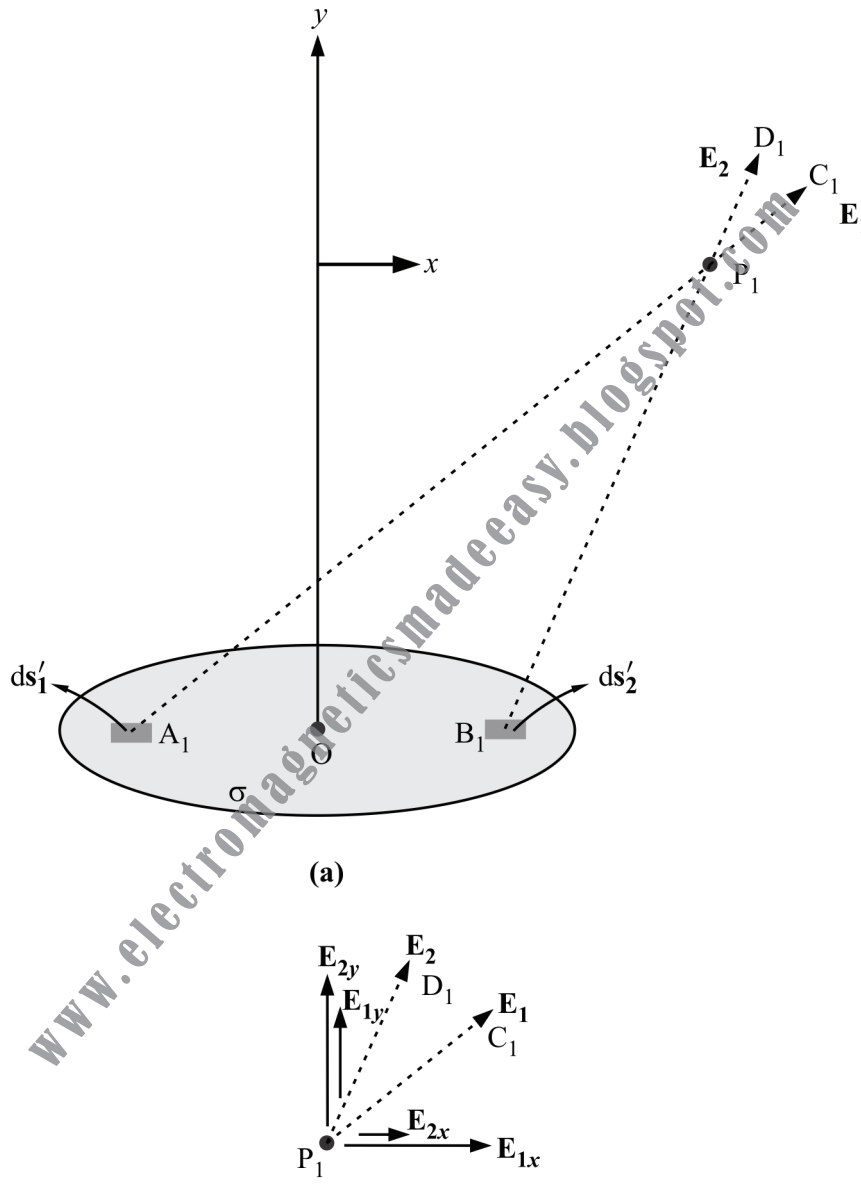


**Fig.2.44**

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direction from $P_1$ to $D_1$. Hence you have to resolve $\mathbf{E_1}$, $\mathbf{E_2}$ into components then add the $x$, $y$ components separately and then finally integrate it to find the field at $P_1$ due to the entire disc. In figure 2.44a, the problem is two dimentional what about three dimentions.

The problem becomes even more complicated in three dimensions because there are now three components to work with $E_x$, $E_y$, $E_z$.

Now consider calculating the electric field **E** using potential formulation. As shown in figure 2.45 we need to calculate the potential at point $P_1$ without worrying about ***"what is the direction of the potential"*** because potential being a scalar doesn't have direction. One need not get confused with components whenever working with V because potential being a scalar doesn't have ***"direction and hence components".***

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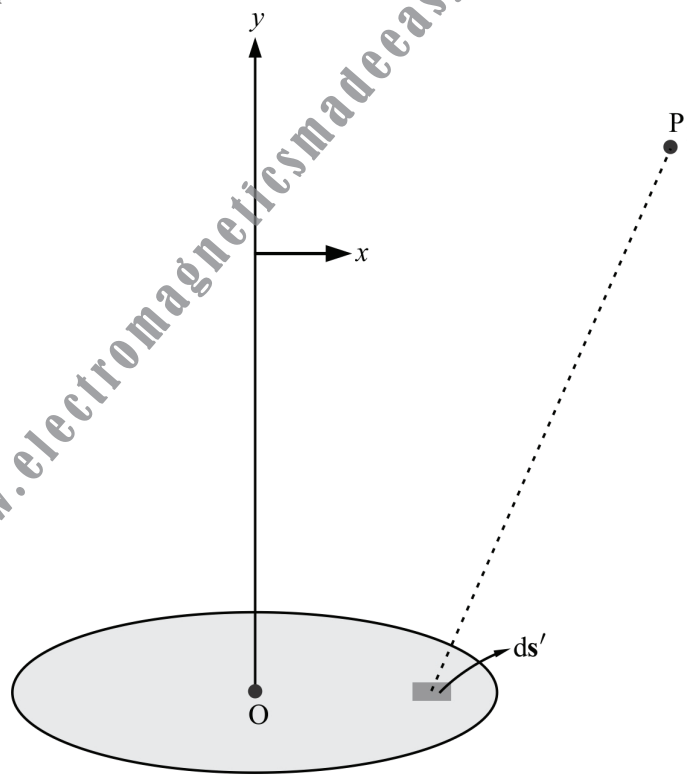


**Fig.2.45**



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Thus if we are going to calculate **E** using Coulomb's law then we have to keep track of the direction and mess up with components. ***"If the charge distribution is complex it is very hectic to calculate E using Coulomb's law because in that case working with components will be highly confusing"***.

In calculating potential V we need not bother about direction because V is a scalar quantity. Once V has been calculated then we can calculate **E** using $\mathbf{E} = -\nabla V$.

In summary we attempt to find **E** for a given charge distribution by using Gauss's law. However when Gauss's law cannot be used then we calculate **E**, first by calculating potential V. Coulomb's law is rarely used.

Finally we have developed three different methods to calculate the electric field of the given charge distribution. Given the initial charge distribution we plug in charge distribution into the respective equations and finally find the electric field. However in most practical cases the "initial charge distribution" is not known. Without even knowing the initial charge distribution we are supposed to find the electric field.

Two methods have been developed to calculate electric field in such situations. Method of images and Laplace's equation method. Before discussing about these methods we will discuss about some intemediate topics like uniqueness theorem, boundary conditions etc., which we will be using in the above methods.

## 2.19 Electric Field of a Dipole

We will calculate the electric field of the dipole in this section. We will be using the concept of dipole in future sections.

Two charges of equal magnitude and opposite sign separated by a small distance form a dipole.

Figure 2.46a shows a dipole in which there are two charges q and –q separated by a small distance d.

We are interested in calculating the electric field at point P in figure 2.46a due to the dipole. It is assumed that the point P is situated at a large distance from O so that d <<< r.

As said in section 2.18 out first attempt will be to use Gauss's law to calculate the electric field. First step is to plot the electric field of the dipole as shown in figure 2.47. Clearly there is no symmetry in the problem.



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